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ABSTRACT

In one project phase, assessments of cognitive levels in grades seven and eight on fractions and ratio concepts were made and compared to the corresponding cognitive demands of curriculum guide objectives, authorized textbooks, teacher presentations, and teacher-made tests. Considerable gaps were found between student cognitive level and curricular demand in the context of fractions and ratios for both grades. Another phase of this project analyzed the effects of a concrete process-oriented approach to the teaching of fractions and ratios in seventh and eighth grades. Overall, and particularly with seventh graders, project findings demonstrated that a concrete, process-oriented approach can result in significantly improved achievement in and attitude toward fractions and ratios. Simultaneously, the development of general mathematical strategies is enhanced and computational facility is maintained. (Author/MP)

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FINAL REPORT 1980

CALGARY JUNIOR HIGH SCHOOL MATHEMATICS PROJECT

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"My own view is that progress in the development of pupils' capacities in these general processes of mathematics is most likely to come from teaching based on pupils' investigation of problem situations, selected and arranged so as to foster the learning of coherent bodies of mathematical ideas, with interactions among pupils in the group, and with the teachers' intervention mainly taking the form of guiding by general questions which it is intended that the pupil will eventually take over as strategies."

A.W. Bell, "The learning of Process Aspects of Mathematics," in Proceedings of the Second International Conference for the Psychology of Mathematics Education, E. Cohors-Fresenborg and I. Wachsmuth, Editors, Osnabruck, 1978.

ABSTRACT

In one phase of the Calgary Junior High School Mathematics Project (CJHMP), assessments were made of student cognitive levels in grade seven and eight fraction and ratio contexts and of the corresponding cognitive demands made by curriculum guide objectives, authorized textbooks, teacher presentations and teacher-made tests. Of the 435 grade seven students tested in the context of fractions, 55.8 percent were at the concrete level, 38.2 percent were transitional, and only 6.0 percent were formal. For grade eight (312 students), the figures were 44.5 percent concrete, 43.3 percent transitional, and 12.2 percent formal. In the treatment of fractions by regular textbooks, teachers and tests in the two grades, the cognitive demands were found to conform to the concrete level less than 4 percent of the time and to the formal level more than 68 percent of the time. Similar patterns were found for ratios. These findings indicated a considerable gap between student cognitive level and curricular demand in the contexts of grade seven and eight fractions and ratios.

Another phase of the CJHMP analyzed the effects of a concrete process-oriented approach to the teaching of fractions and ratios in seventh and eighth grades. The effects measured included achievement in fractions and ratios as well as attitude and ability to apply general mathematical strategies. Six teachers taught 246 grade seven students in ten experimental classes while 234 grade seven students in nine classes were taught by four teachers using regular classroom teaching methods. At the grade eight level, 120 students in four experimental classes were taught by two teachers while 213 students in eight regular classes were taught by four teachers. The students had been matched on standardized school-administered tests. All 813

students were pre-tested on achievement in fractions and ratios, attitude toward fractions and ratios, and general mathematical strategies. Both groups of grade seven students were taught fractions and ratios for approximately twelve weeks. For the grade eight groups, the length of treatment was approximately eight weeks. At the completion of each unit all of the treatment groups were retested using the same achievement and attitude instruments as those administered initially.

Two-factor repeated measures analyses of variance were used to compare treatment groups on attitude towards fractions and ratios (anxiety and enjoyment) as well as the ability to apply general mathematical strategies. Significant differences were found at the 0.05 level favouring the grade seven experimental group on all of these measures. However, at the grade eight level, no significant differences were found between the attitude scores or overall general mathematical strategy scores of the two treatment groups.

The fraction and ratio achievement scores were analyzed using three-factor repeated measures analyses of variance, with treatment group, cognitive level, and test occasion (repeated) as factors. The results of the analyses of the grade seven scores favoured the experimental group with a level of significance of 0.05 on the fractions, ratios, and "fraction problems" measures of improvement. The grade eight findings were similar except that there were no significant differences between the experimental and regular groups in improvement in ratio scores. A significant interaction effect at the 0.05 level between the treatment groups and cognitive levels was found in the grade seven fraction scores but not in those of grade eight. There were no significant differences between the treatment groups in "computation with fractions" scores at either grade level. A priori

comparisons using t-tests at each of the grade seven cognitive levels indicated that there was a significant difference, favouring the experimental group, between the mean increases in fraction scores attained by the CPO and REG transitional level students, but no significant differences were found at the concrete or formal levels. The grade eight t-tests indicated a significant fraction score difference at the formal level favouring the experimental group. The t-test analysis of the grade seven ratio test score increases showed significant differences at the 0.05 level favouring the experimental group at both the transitional and formal levels, but no significant differences were found at the grade eight level.

Overall, but particularly at the grade seven level, the project findings have demonstrated that a concrete, process-oriented approach can result in significantly improved achievement in, and attitude towards, fractions and ratios while enhancing the development of general mathematical strategies and maintaining computational facility.

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OUTLINE OF THE STUDY

Introduction

The reports of the National Assessment of Educational Progress (NAEP) in the United States have indicated that the computational skills of thirteen and seventeen-year-olds are adequate but their problem solving skills are poor, especially when several steps are required to arrive at a solution or when non-routine applications are involved. There is a real danger that, under current "back-to-basics" emphases, skills of number and symbol manipulation may be stressed so heavily that the already serious inability of many students to successfully handle multi-step, non-routine problems will be further intensified. Answer-getting through straight recall and the application of a one-step algorithm have been found to be the predominant modes for approaching problems at all of the age levels tested by the NAEP (Carpenter, et al, 1975). The use of memorized rules rather than reasoning in mathematics appears to be at the heart of the difficulty. Students who know facts and algorithms but have not learned to apply them in novel situations eventually end up in a predicament like that of the fifteen-year-old who remarked that he can do any problem (of the kind he has been working on) if he is given a worked-out example, but on exams he frequently doesn't understand what a question is asking for (evidently neither really understanding the terminology nor having the basic concepts firmly anchored in real experience).

Why are student mathematical concepts and skills not more substantially in evidence at the secondary school level? Analyses of junior high school mathematics programs have indicated that little attention has been given to the development of general mathematical strategies such as drawing conclusions from mathematical investigations, making generalizations, and

giving explanations and proofs (Bell, 1976, p. 5.1).

An underlying assumption in the planning of the Calgary Junior High School Mathematics Project (CJHMP) was that improvement in mathematical achievement is more likely to result from encouraging students to employ general strategies in mathematics by engaging them in a small number of concretely based investigations rather than having them do a large number of routine textbook-type exercises and "problems." The concrete base for the investigations has been prompted by previous small scale studies that have indicated that relatively few grade seven or eight students are at a formal level in the context of ratios (e.g., Onslow 1976). What proportions of Calgary grade seven and eight students are at formal, transitional or concrete levels in the contexts of ratios and fractions? What are the cognitive demands of the curriculum? Can a concretely-based, process-enriched teaching approach be shown to provide benefits beyond those of the regular approaches? Questions like these helped to shape the CJHMP design and focus.

The experimental teaching methods used in the project have been modelled on the pioneering work of the South Nottinghamshire Project (SNP). The SNP teaching approaches characteristically use a variety of simple concrete materials to pose "well-motivated" problems in which the concrete "props" facilitate understanding the mathematical ideas involved (Bell, 1976, p. 5.4). A pupil carries out a mathematical investigation beginning with concrete materials, experimenting, recording what happens, formulating questions, and writing-up accounts of experimental results as well as applying the results to practical situations. Such strategies are not generally embodied in North American school mathematics textbook materials nor do they occur with any frequency in the most common secondary school mathematics mode

of instruction: "example-rule-exercises." Given that the need for learning general mathematical strategies is real, could North American mathematics teachers be led to successfully use process-enriched, student-investigation teaching approaches? What effects would a process-enriched teaching approach have on student achievement, attitude, and application of general mathematical strategies?

Purpose

In the contexts of "ratio and proportion" and "rational numbers" at the grade seven and eight levels, the purpose of the study was to assess: student cognitive abilities, current curriculum cognitive demands, curricular cognitive demands as interpreted in the classroom, and the effects of using a process-enriched instructional method of teaching that is adaptable to different student levels of cognition.

Problems Examined

1. What levels of cognitive development are demonstrated by students in grades seven and eight in the contexts of rational numbers and ratio and proportion?

2. What are the cognitive demands of the present Alberta school mathematics curriculum in the contexts of rational numbers and ratio and proportion as indicated by:
   a. the 1978 Alberta Junior High School Mathematics Curriculum Guide
   b. currently authorized textbooks
   c. teacher presentations
   d. teacher-made tests?

3. How well do the cognitive demands of selected aspects of the curriculum fit the cognitive abilities of the majority of the students?

4. Does the use of teaching strategies that are adaptable to student cognitive levels significantly improve achievement in and attitude towards rational numbers and ratio and proportion topics?

The Scope of the Study

Recent information about student cognitive development at the seventh and eight grade levels contributed to the choice of grades for the project, as did the fact that, in the Alberta Curriculum, the longest treatment of fractions and ratios occurs in these grades, but the major considerations were that fractions and ratios are seen as the most difficult and among the most important topics in the mathematical development in grades seven and eight and that these topics have been extensively studied with respect to cognitive development level.

The study was restricted to a set of schools whose classes had been matched on the basis of the Stanford Intermediate Level II, Form B, test results and whose principals and teachers were agreeable to the conduct of the study.

There were 246 seventh grade students in ten classes taught by six teachers in the experimental or concrete process-oriented method (CPO) and a total of 234 students in nine classes taught by four teachers in the grade seven control or regular classroom method (REG). In the eighth grade experimental group there were 120 students in 4 classes taught by 2 teachers,

and in the corresponding regular group there were 213 students in 8 classes taught by 4 teachers.

Definitions of Terms

CPO refers to the Concrete Process-Oriented teaching method. This experimental method, modelled after the South Nottinghamshire Project teaching material (Bell, Wigley and Rooke, 1978, 1979), leads students to use concrete manipulative material to investigate problems in fractions and ratios, recording and discussing the results of these investigations.

REG refers to the Regular Classroom teaching method. The regular teachers normally taught fractions and ratios using an example-rule-exercises approach.

Concrete, used to describe a cognitive level or demand in the present study, refers either to the ability to handle fraction situations involving: taking one-half or one-third of a set of objects, a drawing, or a number; doubling or tripling to produce equivalent fractions; or adding or subtracting with physical models; or to the ability to handle ratio and proportion situations involving; 1 to 2, 1 to 3, or their reciprocals; only doubling or tripling or the reverse; or action with countable real objects (a more complete description of the concrete criteria is given in the Appendix).

Transitional, used to describe a cognitive level or demand in the present study, refers to the ability to handle fraction or ratio and proportion situations involving: pictorial support containing countable parts, or abilities beyond the concrete but not formal (see Appendix for more detail).

Formal, used to describe a cognitive level or demand in the present

study, refers either to the ability to handle fraction situations involving: at least one element greater than 4; comparisons made in terms of different sized units; second order thinking; symbolic representations of fractional relationships and operations; or expectation of a correct explanation of procedures; or to the ability to handle ratio and proportion situations involving: at least one component greater than 4; second order proportional thinking; symbolic representations of proportional relationships; or explanation using proportional reasoning (see Appendix for more detail).

Research Methodology

Towards the end of August, 1979, the experimental teachers were presented with process-enriched material from various sources which covered the topics in fractions and ratios as outlined by the Alberta Education Curriculum Guide for grades seven and eight. Several investigations were tried out with the experimental teachers, and teaching strategies were discussed along with ideas for adapting the materials and methods to local junior high mathematics classrooms. The teaching material for the CPO method presented students with concrete material that they could use to investigate ideas in fractions and ratios to build up their own concepts without being formally told by the teacher what the main ideas were. After trying out and discussing the material with the investigators, the teachers selected material that they considered appropriate for the teaching of fractions and ratios in grade seven and eight. How the CPO approach fared as observed in classroom implementation is described under the Findings of the Study.

During September, 1979, tests were administered to all participating students to assess their cognitive levels in ratios (as measured by Onslow's

Ratio and Proportion Test) and in rational numbers (as measured by Kieren's Fractional Number Thinking Test, which was developed specifically for the CJHMP). Percentage distributions of students by cognitive level in fraction (rational number) and ratio contexts were compiled for comparison with the percentage distributions of curricular cognitive demands.

Also during September, the students in the treatment and control classes were pre-tested with an achievement test battery: CSMS Fractions, CSMS Ratios, and Bell's General Mathematical Strategies test, as well as a pair of attitude tests: Attitude towards Fractions and Attitude towards Ratios.

To address the question of the cognitive demand levels of the rational numbers and ratio and proportion sections of the Alberta Junior High School Mathematics Curriculum Guide, two sets of cognitive demand level criteria were established by inference from Onslow's Ratio and Proportion Test, Kieren's Fractional Number Thinking Test, and related research literature. The criteria (copies of which are included in the Appendix) were used to determine the percentage of concrete, transitional and formal items in the treatment of grade seven and eight rational number and ratio topics by the curriculum guide, authorized textbooks, teacher presentations, and teacher-made tests.

The Concrete Process-Oriented (CPO) teachers taught the Concrete Process-Oriented (CPO) units to one or more of their classes as part of their regular teaching load for the time period normally suggested by the Calgary Board of Education Mathematics Team (eleven and a half weeks for grade seven, seven and a half weeks for grade eight). For roughly the same period of time, the Regular (REG) teachers taught the same topics in the usual manner following one of the currently authorized textbooks. While the fraction and

ratio units were being taught, classroom observations were made to document some of the characteristics of the CPO and REG methods as used in the classroom. Further information on the two methods was collected when the students completed a questionnaire designed to record their perceptions of the use of materials, of student investigations, and of various teacher presentations. As each ratio or fraction unit was completed the classes were retested with the relevant achievement and attitude tests. The final test administered to all classes at the conclusion of the teaching of fractions and ratios was the General Mathematical Strategies test.

To assess improvement in achievement in fractions and ratios, the statistical analysis used was a three-factor, repeated measures analysis of variance of fraction and ratio test scores. The first factor was treatment group (CPO or REG), the second factor was cognitive level in the context of fractions or ratios, and the third factor was test occasion (prior to or after the treatments).

The data from the Attitude towards Fractions and Attitude towards Ratios subscales, Anxiety and Enjoyment, and the General Mathematical Strategies Test and subscales were analyzed using a two-factor repeated measures analysis of variance. The first factor was treatment group (CPO or REG) while the second factor was test occasion.

Major Sources of Data

The following measuring instruments were administered to the 813 grade seven and eight students who participated in the CPO and REG classes.

1. Onslow's "Ratio and Proportion Test" (a paper-and-pencil simulation of clinical interview classification of students by cognitive level in

the context of ratios and proportions)

2. Kieren's "Fractional Number Thinking Test" (a paper-and-pencil instrument constructed from individual interviews for assessing cognitive ability in fraction contexts)
3. CSMS (Concepts in Secondary Mathematics and Science) "Ratio Test" (developed by the CSMS group at Chelsea College, London, and slightly Canadianized for use in the present study as an achievement test)
4. CSMS "Fractions, Year 1 and 2" test (developed by the CSMS group and slightly Canadianized for use in the present study as an achievement test)
5. Bell's "General Mathematical Strategies" test (developed by A.W. Bell to measure ability to give explanations, to follow instructions, to recognize and express relationships, and to generate examples)
6. "Attitude towards Ratios" scale (a short, Likert-scale measure of ratio enjoyment and anxiety patterned after Aiken's "Attitude towards Mathematics" scale by S. Brindley)
7. "Attitude towards Fractions" scale (same as the preceding but for fractions)
8. "Student Questionnaire on Ratios" (constructed by S. Brindley with 16 Likert format questions designed to determine whether or not students perceived differences between the experimental and regular classroom teaching methods)
9. "Student Questionnaire on Fractions" (same as the preceding but for fractions)

Open-ended "Feedback Sheets" were also filled in by the students in the experimental classes towards the end of the experimental treatment.

The experimental and regular teachers were asked to fill out a "Teaching Styles Questionnaire, Ratios, Grade 7, 8" and a "Teaching Styles Questionnaire, Fractions, Grade 7, 8", which contained questions having patterns and purposes similar to those of the Student Questionnaires, but worded especially for the teacher and designed to sample a wide range of perceptions with regard to the teaching methods being used. The "Teaching Styles Questionnaires" and "Student Questionnaires" were also administered in a "Survey" of seven classes outside of the treatment classes to determine whether or not the regular teachers' and students' responses were typical.

Supplementary descriptive information was gathered during classroom visits by means of a "CJHMP Classroom Observation Form" constructed around ideas and items drawn from the QUEST Project (Centre for Research in Teaching, University of Alberta and the Edmonton Public School Board) and the SCAN Project (Shell Research Centre, University of Nottingham, England).

Two sets of cognitive demand criteria, "CJHMP Cognitive Demand Levels: Ratios and Proportion" and "CJHMP Cognitive Demand Levels: Fractions," were constructed as previously described (see Appendix). In order to establish levels of cognitive demand in the relevant curricular materials and procedures, the criteria were applied to the curriculum guide, to authorized textbooks, to teacher-made tests, and to the information gathered from classroom observations.

FINDINGS OF THE STUDY

Student Cognitive Levels

The assessment of the grade seven student cognitive levels in fractions showed that 55.8 percent were at the concrete level, 38.2 percent

were at the transitional level and only 6.0 percent of the students were at the formal level. The grade eight student distribution showed 44.5 percent concrete, 43.3 percent transitional, and 12.2 percent formal, in the context of fractions. The grade seven assessment of the student cognitive levels in ratios indicated that the percentage of students in the concrete, transitional and formal levels were 50.2, 43.0 and 6.8 percent, respectively. The grade eight distribution was 35.3 percent concrete, 54.2 percent transitional and 10.5 percent formal, in the context of ratio (See Tables 1 and 2, Figures 1 and 2).

Cognitive Demands

The findings from cognitive demands analyses of the curriculum guide, authorized textbooks, teacher-made tests, and teacher presentations in the contexts of fractions and ratios are also summarized in Tables 1 and 2. For example, at the grade seven level, while 41 to 57 percent of the Curriculum Guide objectives for fractions and ratios are formal, 55 to 77 percent of the cognitive demands in regular classrooms or materials were found to be at a formal level (as represented by Teacher Presentation (REG), Teacher-made Tests, and Textbooks). In grade eight fractions and ratios, the proportion of formal demands ranged from 0 to 100 percent (as compared with 73 to 80 percent formal Curriculum Guide objectives). Just as, in general, a greater proportion of formal demands was being made in regular classrooms than is implied by curricular objectives, there were fewer concrete demands being made, as compared with the objectives. For example, with the exception of the Introduction to Ratio sections in the grade seven textbooks (20 percent concrete), the classroom interpretations of the Curriculum Guide made cognitive demands at the concrete level either not at all or less than 6 percent of the time (as compared with 21 to 28 percent of the grade seven, and

none of the grade eight, Curriculum Guide objectives implying concrete level demands).

The cognitive demand criteria used in the analyses are summarized in the Definitions of Terms section, and a more detailed description is given in the Appendix. The two textbook series for which average cognitive demands ratings are given in Tables 1 and 2 are: Holt Math 1 and 2 and Math Is 1 and 2. In the Tables, "items" refers to the number of objectives, questions, or examples contained in the material analyzed. Similarly, "minutes" refers to the number of minutes of class time analyzed.

Cognitive Demands vs Abilities

If some proportion of students in a given class is operating at a concrete level in a topic being studied, it seems reasonable that a comparable proportion of class time and class work would be provided to make demands at a similar level. Accordingly, in order to test the "goodness-of-fit" between the distribution of students by cognitive level and the various cognitive demand distributions, non-standard chi-square analyses were used. The relevant percentage distributions and the chi-square values obtained are given in Tables 1 and 2. While all of the cognitive demand distributions proved to be significantly different from the relevant student cognitive level distributions, it appears that the Curriculum Guide and CPO Teacher Presentation distributions showed a better fit than the others with the Student Cognitive Level distributions. The Textbooks and Teacher-made Tests were consistently relatively heavily weighted with formal level demands. Figures 1 and 2 display the percentage distribution data from Tables 1 and 2 in a graphical form.

TABLE 1

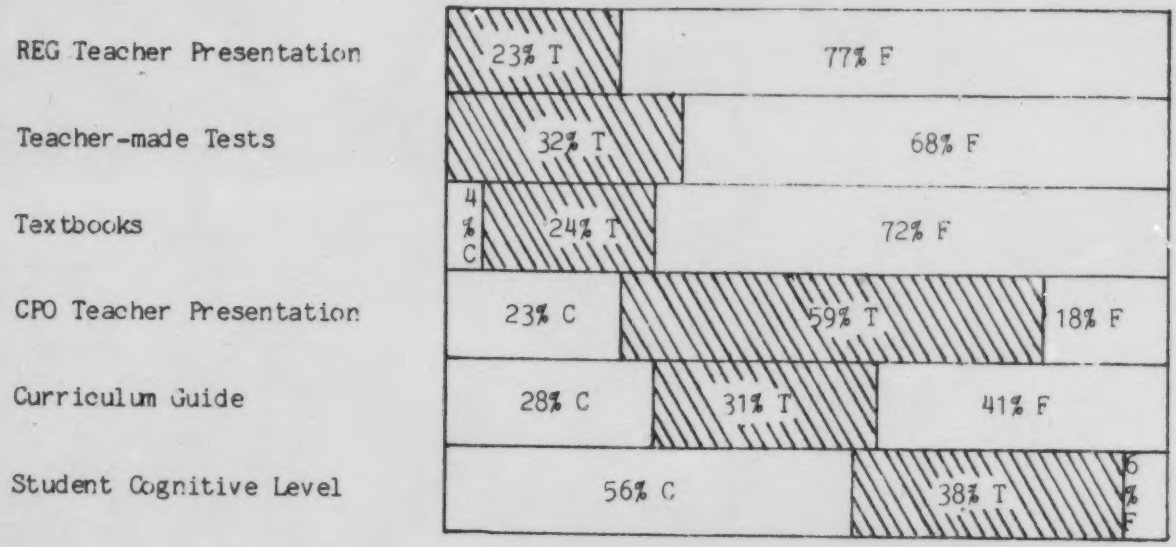
COGNITIVE DEMANDS AND STUDENT COGNITIVE LEVELS

FRACTIONS (RATIONAL NUMBERS)

Grade 7	N	Concrete	Transitional	Formal	Chi-square*
Grade 7 Curriculum Guide	29 items	28%	31%	41%	45.99
Textbooks (2), Fractions Introduction	385 items	4%	24%	72%	434.04
Teacher-made Tests (2),REG	57 items	0%	32%	68%	181.17
Teacher Presentation CPO	393 minutes	23%	59%	18%	96.59
REG	70 minutes	0%	23%	77%	235.79
Student Cognitive Level (Fractions)	435 students	56%	38%	6%	
Grade 8					
Grade 8 Curriculum Guide	11 items	0%	27%	73%	32.88
Textbooks (2),Fractions	412 items	3%	18%	79%	346.24
Teacher-made Tests (2)	56 items	0%	2%	98%	186.12
Teacher Presentation CPO	120 minutes	22%	75%	3%	33.90
REG	40 minutes	0%	100%	0%	45.64
Student Cognitive Levels (Fractions)	312 students	45%	43%	12%	Critical (0.01, 2df) Chi-square: 9.21

* Chi-square test: "Goodness of fit" with Student Cognitive Level Distribution

Grade 7 Fractions (Rational Numbers)



Grade 8 Fractions (Rational Numbers)

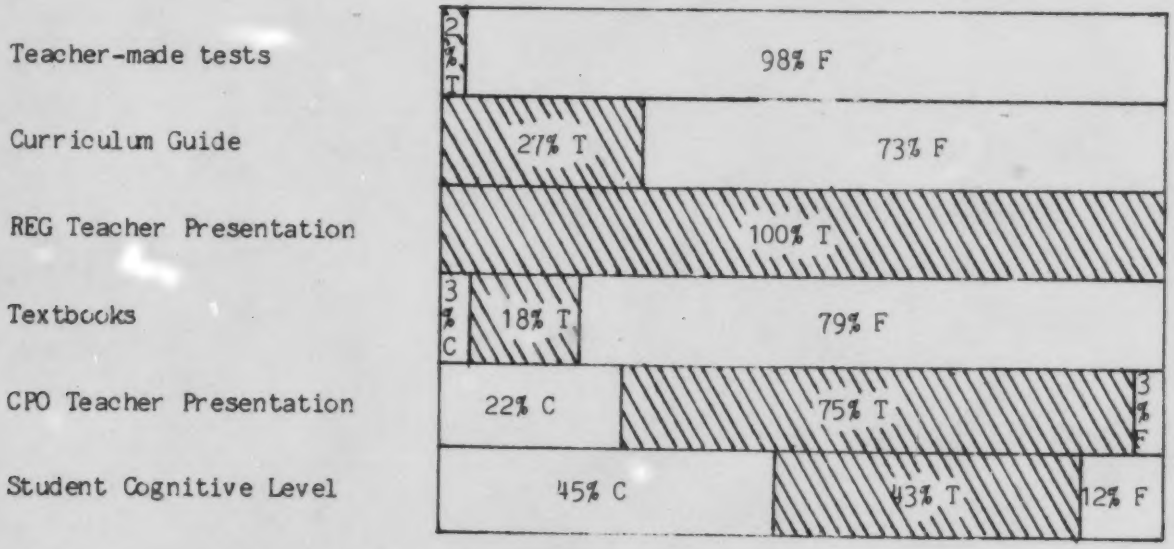


FIGURE 1
CURRICULAR COGNITIVE DEMANDS CONTRASTED WITH STUDENT COGNITIVE ABILITY LEVELS IN FRACTION CONTEXTS

* Table 1 contains additional details C- Concrete; T- Transitional; F- Formal

TABLE 2

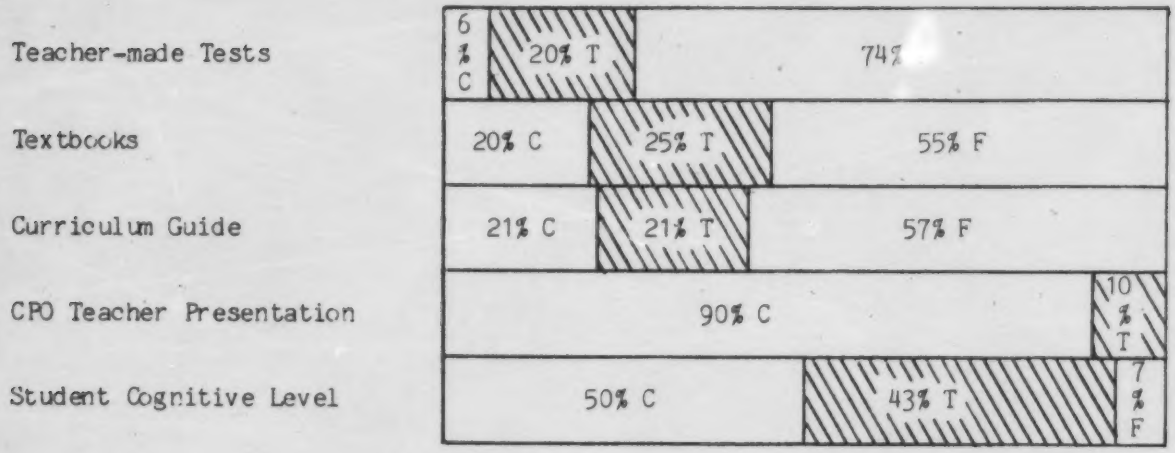
COGNITIVE DEMANDS AND STUDENT COGNITIVE LEVELS

RATIO AND PROPORTION

Grade 7	N	Concrete	Transitional	Formal	Chi-square*
Grade 7 Curriculum Guide	14 items	21%	21%	57%	45.14
Textbooks (2),Ratio Intro.	249 items	20%	25%	55%	207.68
Teacher-made tests (2),REG	35 items	6%	20%	74%	145.03
Teacher presentation CPO	105 minutes	90%	10%	0%	54.19
REG	(Not observed)				
Student Cognitive level (Ratios)	455 students	50%	43%	7%	
Grade 8					
Grade 8 Curriculum Guide	5 items	0%	20%	80%	16.76
Textbooks (2), Ratios	194 items	1%	9%	91%	318.92
Teacher-made tests (2),REG	38 items	3%	5%	92%	143.84
Teacher presentation CPO	62 minutes	16%	77%	6%	11.61
REG	40 minutes	0%	0%	100%	172.48
Student Cognitive Level (Ratios)	312 students	35%	54%	11%	Critical (0.01, 2df) Chi-square: 9.21

* Chi-square test: "Goodness of fit" with Student Cognitive Level distribution.

Grade 7 Ratio and Proportion



Grade 8 Ratio and Proportion

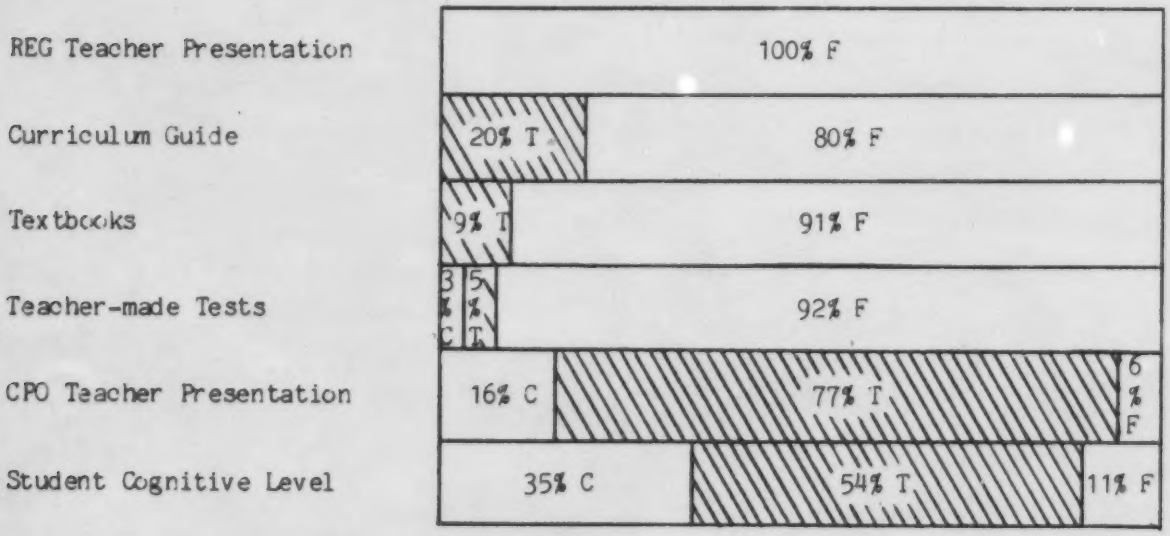


FIGURE 2

CURRICULAR COGNITIVE DEMANDS CONTRASTED WITH STUDENT COGNITIVE ABILITY LEVELS IN RATIO AND PROPORTION CONTEXTS

* Table 2 contains additional details C- Concrete; T- Transitional: F- Formal

Experimental CPO vs Regular REG Teaching Methods

Periodic classroom observations were made to determine whether or not the experimental teachers actually did use teaching methods that were demonstrably different from those used by the regular teachers. It appeared that the CPO teachers were conscientiously following the material and procedures discussed during the preparation week in August. However, they did find the method somewhat difficult to adapt to in that it was seen as quite different from the approaches to which the students were accustomed. Having students work at investigations in groups of four was tried in the first few weeks of the experiment, but the teachers felt that there was too much "social" interaction rather than mathematical interaction occurring and, subsequently, such groupings tended to be avoided. The experimental method generated more discussion between teacher and pupil than is usually the case, and teachers were disturbed to discover how little students understood of ratio concepts, for example. Because the teachers had less direct control of the teaching of ratio concepts in that they did not have a quiet classroom to address as a whole nor did they have the opportunity to explain a new concept and follow it with examples, they felt very concerned about the amount of learning that was taking place with student investigations. As mentioned previously, there was a general feeling that not enough examples were being completed. The teachers also felt somewhat insecure because this was the first time they had taught complete units using a concrete process-oriented method. With extra noise from talking and Cuisenaire rods dropping, it seemed to them that little was being learned. However, after three weeks of the CPO ratio unit, when teacher-made tests from the previous year were administered by some of the experimental teachers, it was observed that the CPO students

outperformed those of the preceding year.

By the end of the ratio unit, the CPO teachers seemed to have adapted to the CPO method and were more comfortable teaching this way in the fraction unit. The observers often randomly selected a few students from the classes, reading their reports and asking them how they liked the teaching method. Almost invariably the students said that they liked the CPO method better than previous ones. In an informal appraisal students were asked to anonymously indicate on a sheet what they liked most about the method and what they liked least. Almost without exception the answers given indicated that they liked the investigations (such as those with the Cuisenaire rods, for example) but they did not like writing reports about what they had found in an investigation.

It was difficult for the CPO teachers to adapt to the method of letting students find mathematical patterns, concepts and procedures without telling them, because they were accustomed to giving direct explanations. They tended to feel uncomfortable watching a student struggle with a concept by himself. Asked if they would use this method again after the completion of the investigation, most teachers indicated that there were some parts of the method that they really liked and would use the next time they taught fractions and ratios. Subsequent to the Project classroom treatment phase, two of the CPO teachers had an opportunity to edit the CPO materials with the assistance of seven teachers from outside of the Project. This group produced a "Grade Seven and Eight Investigation Manual" consisting of 15 "Fraction and Decimal" and 6 "Ratio and Percent" investigations. The format used was patterned after those of science labs. The approach tended to be somewhat less open ended than the original CJHMP material but seemed to retain the rest

of the essentials.

The Regular (REG) classrooms were found to be fairly quiet with students doing examples from the book and the teacher helping where necessary. The Regular and "Survey" teachers' responses to the "Teaching Styles Questionnaire" consistently affirmed that the most frequent teaching pattern consists of: doing several examples on the board, carefully and clearly explaining the rules used, assigning similar problems to the students, and working with individuals experiencing difficulties.

An analysis of the student questionnaires indicated a significant difference between the experimental and regular class teaching methods as perceived by the students. One question, for example, which referred to being told a rule before attempting a problem, showed a significant difference between experimental and regular classes, with this happening much less frequently in the experimental classes. The analyses also showed that students in the experimental classes more frequently used manipulatives for investigating ideas in fractions and ratios. They were also more frequently encouraged to invent their own solutions to problems, to notice number patterns, to work in pairs or groups, and to write reports of what they had found out while investigating fraction and ratio problems. Also more frequently in evidence in CPO classes were: group discussion, checking the correctness of solutions, and meeting together to discuss the correctness of results. The questions that were not responded to in a significantly different manner by the two groups were those concerning the use of and giving of rules. There was a very close correspondence between the REG student, REG teacher, "Survey" student, and "Survey" teacher responses to the same questions on the Teaching Styles Questionnaire, indicating a consistency in

the way students and teachers perceive the regular teaching approaches for ratios and fractions (see Tables 7 and 11 and the Teaching Styles Questionnaires in the Appendix).

During the classroom visits the observers filled out a "CJHMP Classroom Observation Form," a copy of which is included in the Appendix. An observer or, on occasion, two observers, would sit quietly at the back of the class being observed and would fill in codes and/or comments in relation to one-minute intervals of elapsed class time on the "CJHMP Classroom Observation Record." The codes indicated what kind of grouping, teaching method, materials, teacher questions, and cognitive demands were observed, and the positioning of the codes on the sheet indicated how long they lasted. Subsequent to the lesson, the "CJHMP Behaviour Frequency Record" was filled in with a short written summary of the lesson content and Likert-scale ratings of the frequency with which twenty-one different classroom behaviours were observed. Tables summarizing what was recorded on the experimental and regular classroom observation forms are included in the Appendix, but the highlights from the overall patterns observed are presented in the following descriptions and in Figure 3.

1. Grouping. In the regular classes it was found that, on the average, 52% of the class time was spent in a whole class format (presentation and/or discussion) and, during the remaining 48%, students worked individually at seatwork. In the experimental classes, on the average, 39% of the time was spent in whole class formats, 35% in individual seatwork, 18% in pairs, and 3% in small groups.
2. Method. On the average, regular class students spent 48% of the time

working on textbook exercises as compared with 7% in the experimental classes. Teacher exposition to groups of 5 or more averaged 26% of the time in regular classes and 8% in experimental classes. Dialogue between the teacher and groups of fewer than 5 students averaged 8% of the time in the regular and 19% in the experimental classes. None of the observed regular classes incorporated real objects into the lessons, but, in the experimental classes, students worked and investigated with real objects 31% of the time, on the average.

3. Materials. Textbook exercises and worksheets were used 48% and 21% of the time in the regular classes while in the experimental classes the percentages were 18% and 15%, respectively. According to the coded records made during the classroom observations, the experimental class students had access to concrete materials for 31% of the class time, but no concrete materials were observed in use in the regular classes. In the regular classes, pictorial material was used in the context of teacher demonstrations 18% of the time. Pictorials were only used 7% of the time in the experimental classes, but they were handled by the students themselves.

4. Teacher Questions. It was observed that 33% of the regular teacher questions versus 11% of the experimental were of a straight recall type requiring no processing (single fact, single act). Simple exercise questions (putting together several facts or acts) made up 73% of the experimental teacher questions and 59% of the regular. Questions involving new ideas and extensions of previous work accounted for 16% of the questions in the experimental classes and 8% in the regular classes.

5. Cognitive Demand. The percentage distributions of the cognitive demands made by the experimental (CPO) and regular (REG) Teacher Presentations are presented by grade level and topic (fractions or ratios) in Tables 1 and 2. Overall, in the context of grade 7 and 8 fractions and ratios, the cognitive demands made by the experimental teachers averaged 15% at the concrete level, 68% transitional, and 17% formal while the regular teacher demands averaged 0% concrete, 35% transitional, and 65% formal.

6. Behaviour Frequency. The use of real objects when learning new ideas and while investigating fraction and ratio problems occurred with moderately high frequency in the experimental classes but was not observed in the regular classes (Items 1 and 2 on the CJIMP Behaviour Frequency Record). Similarly, student writing of reports resulting from small group investigations of fraction and ratio problems was observed with moderately high frequency in the experimental classes but was not observed in the regular classes (Item 7). The experimental teachers demonstrated a moderately high frequency of relating independent activities and investigations to concepts being learned as well as providing for a gradual transition from concrete to more abstract activities; the frequency of such activities in the regular classrooms was low (17, 18). In the regular classrooms there was a moderately high frequency of students being given rules and/or examples before problems were attempted or new work started, while such observations had a low frequency in experimental classes (8). Both regular and experimental teachers displayed similar frequency patterns in using systems for spot-checking student assignments and in

providing evidence of caring, accepting, and valuing student responses (16, 19). A summary of the average frequency ratings from the regular and experimental classroom observations for the items so far described, as well as for the remaining ones on the CJHMP Behaviour Frequency Record, are given in the Appendix.

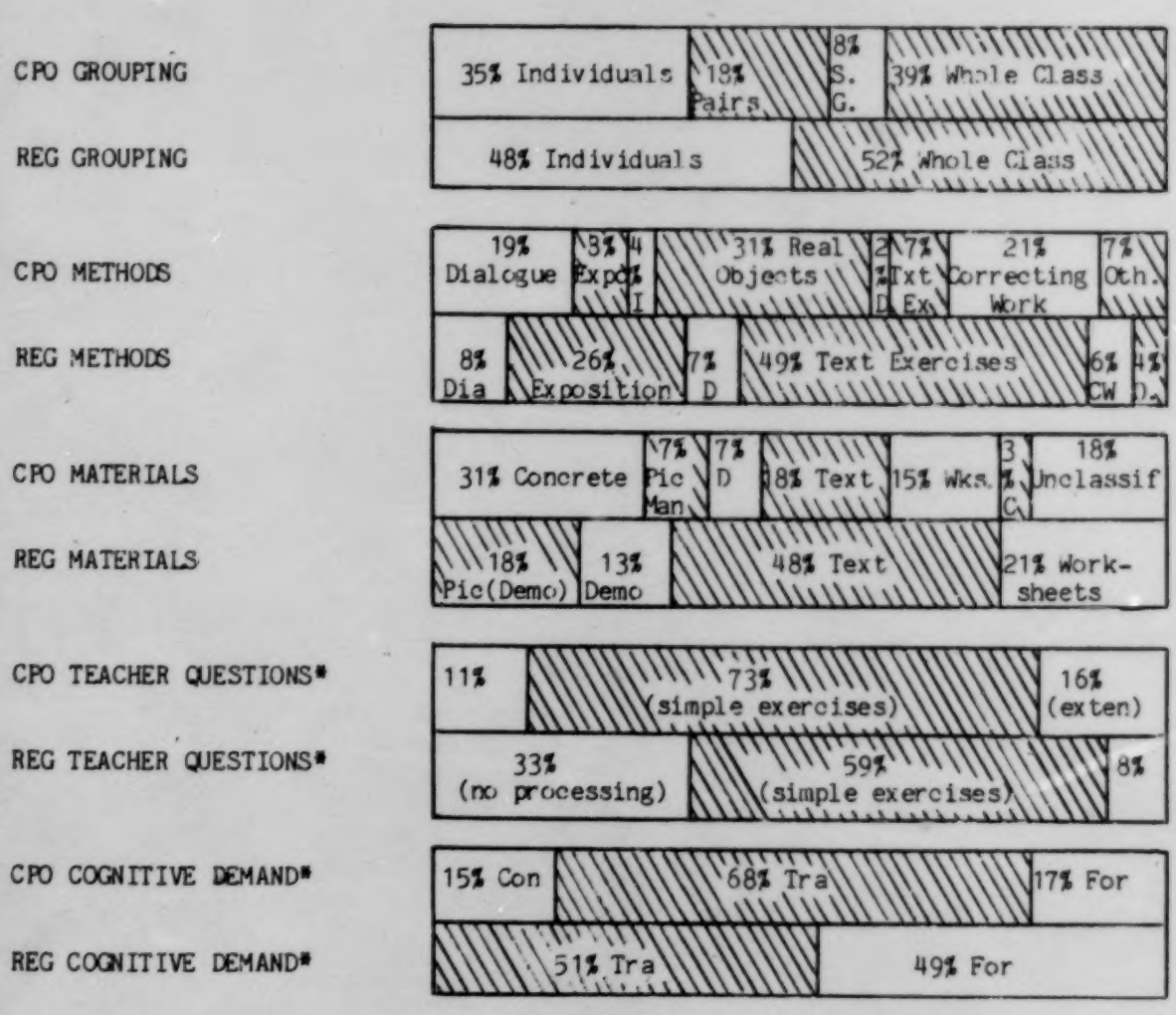


FIGURE 3

AVERAGE PERCENTAGES OF OBSERVED CPO AND REG CLASS TIME SPENT IN SPECIFIC MODES

* Table 12 in the Appendix contains more detail.

Abbreviations: C- Chalkboard: Con- Concrete; CW- Correcting Work; De, Demo- Demonstration; Dia- Dialogue; D- Discussion; Exten- Extensions; Expo- Exposition; For- Formal; I- Investigation; Man- Manipulatives; O, Oth- Other; Pic- Pictorial; S.G.- Small Groups; Text Ex- Text Exercises; Tra- Transitional; Unclassif- Unclassified; Wks- Worksheet.

Grade 7 Achievement: Experimental vs Regular

The statistical analyses results presented in Table 3, demonstrate that, for achievement in fractions as measured by the pre- and post-test scores on the CSMS Fractions test and on the CSMS Fractions Problem subtest, the CPO group scored significantly higher on the mean measures of improvement from pre- to post-test than did the REG group, at the 0.05 level. It was also observed that significant differences on measures of improvement existed among the different cognitive levels. Furthermore, there was a significant interaction effect at the 0.05 level between groups and cognitive levels, indicating a differential effect between treatment groups and cognitive levels on the CSMS Fraction test and the CSMS Fraction Problem subtest. In a priori t-test comparisons between the two treatment groups, the CPO transitional level mean increases were significantly higher at the 0.05 level, while there was no significant difference observed between the two groups at the concrete and formal levels. Accordingly, in the context of fractions, those students at the transitional cognitive level in the CPO group improved significantly more than either other level on both the CSMS Fraction test and the Problem subtest. Figures 4 and 5 display graphically the salient findings from the Grade 7 fraction achievement score analyses.

There was no significant difference between the mean improvement scores attained by the Grade 7 treatment and control groups on the CSMS Fractions Computation subtest. Significant differences among the mean improvement scores of the different cognitive levels were observed at the 0.05 level. No significant interaction effect between treatment groups and cognitive levels was observed.

The results from the Grade 7 CSMS Ratios achievement test are also

presented in Table 3, and in Figure 6, indicating that the CPO group improved significantly more than the REG group at the 0.05 level on the CSMS Ratio test. No significant differences among the cognitive levels were observed, and there was no significant interaction effect between groups and level.

A priori t-test comparisons between the two groups at each cognitive level revealed that the CPO group improved significantly more than the REG group at the transitional and formal levels.

TABLE 3

CPO VERSUS REG MEAN CONTRASTS WITH PROBABILITY LEVELS FROM REPEATED MEASURES ANALYSES OF VARIANCE ON CSMS FRACTIONS AND SUBSCALES AND CSMS RATIOS SCORES. GRADE 7

GROUP		CPO			REG			Probability
		Cognitive Levels			Cognitive Levels			
Test or Subtest		Conc.	Tran.	Form.	Conc.	Tran.	Form.	
CSMS Fractions	N	108	76	13	85	67	10	
	Pre-	27.9	43.6	56.7	25.4	44.1	61.3	Group (G) 0.008
CSMS Fractions	Diff.	13.4	19.0	16.6	12.8	13.5	11.6	Level (L) 0.007
(Max = 84)	Post-	41.3	62.6	73.3	38.2	57.6	72.9	G X L Int. 0.037
CSMS Fractions	Pre-	6.3	13.2	21.5	5.4	13.1	22.0	Group (G) 0.049
(Problem)	Diff.	4.5	8.9	8.2	4.5	5.5	6.9	Level (L) 0.000
(Max = 36)	Post-	10.8	22.1	29.8	9.8	18.6	28.9	G X L Int. 0.014
CSMS Fractions	Pre-	1.8	4.4	7.4	1.2	3.6	7.1	Group (G) 0.550
(Computation)	Diff.	3.3	3.9	2.8	2.8	4.1	2.4	Level (L) 0.004
(Max = 12)	Post-	5.1	8.4	10.2	3.9	7.7	9.5	G X L Int. 0.548
CSMS Ratios	N	98	74	18	68	70	6	
	Pre-	6.8	10.2	16.2	6.5	11.1	19.0	Group (G) 0.001
CSMS Ratio	Diff.	2.5	4.0	4.4	1.6	1.1	1.0	Level (L) 0.145
(Max = 28)	Post-	9.2	14.2	20.6	8.1	12.2	20.0	G X L Int. 0.183

N = Number Tran. = Transitional Int. = Interaction
Conc. = Concrete Form. = Formal

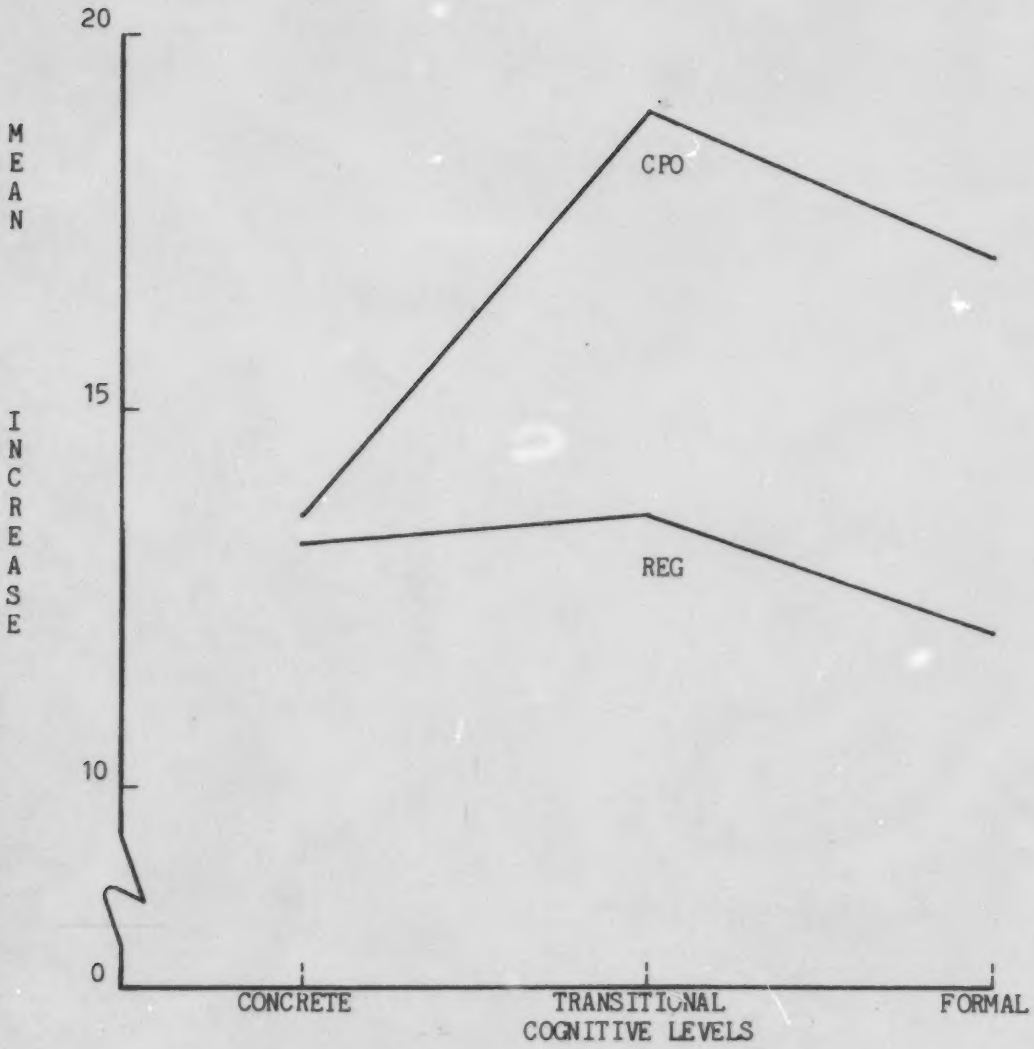


FIGURE 4

MEAN INCREASE IN CSMS FRACTION SCORES

BY TREATMENT GROUP AND

COGNITIVE LEVEL

GRADE 7

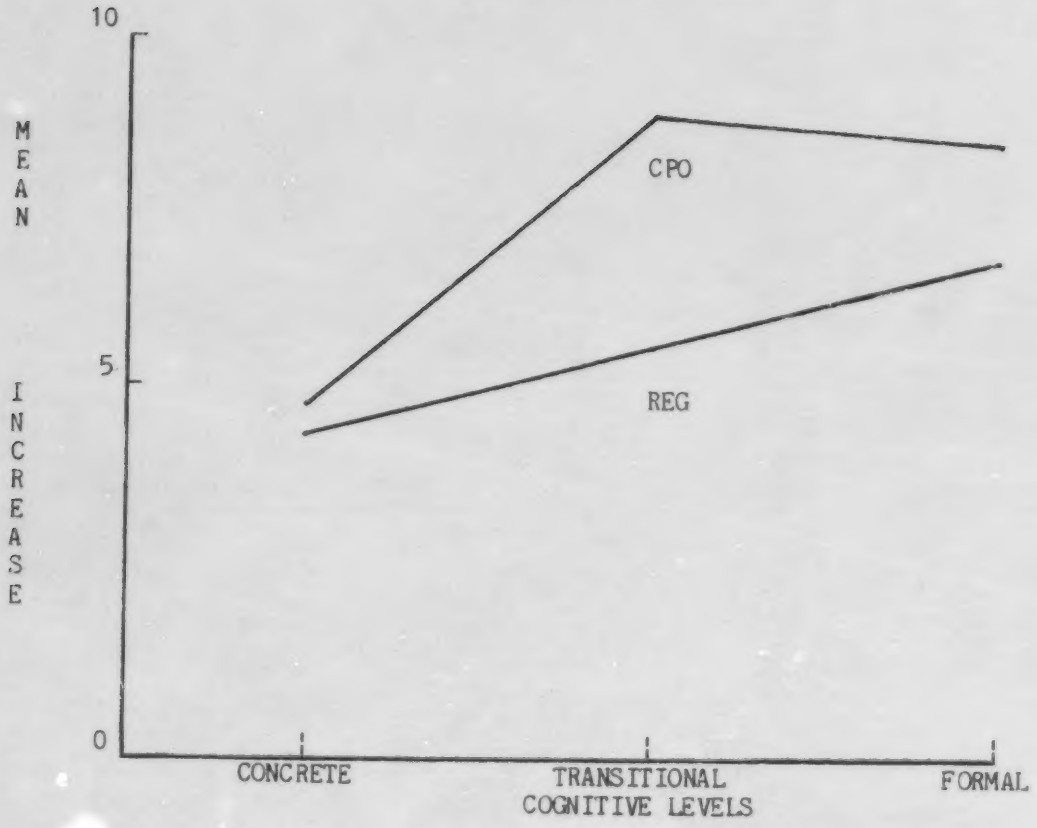


FIGURE 5

MEAN INCREASE IN CSMS PROBLEM SCORES

BY TREATMENT GROUP AND

COGNITIVE LEVEL

GRADE 7

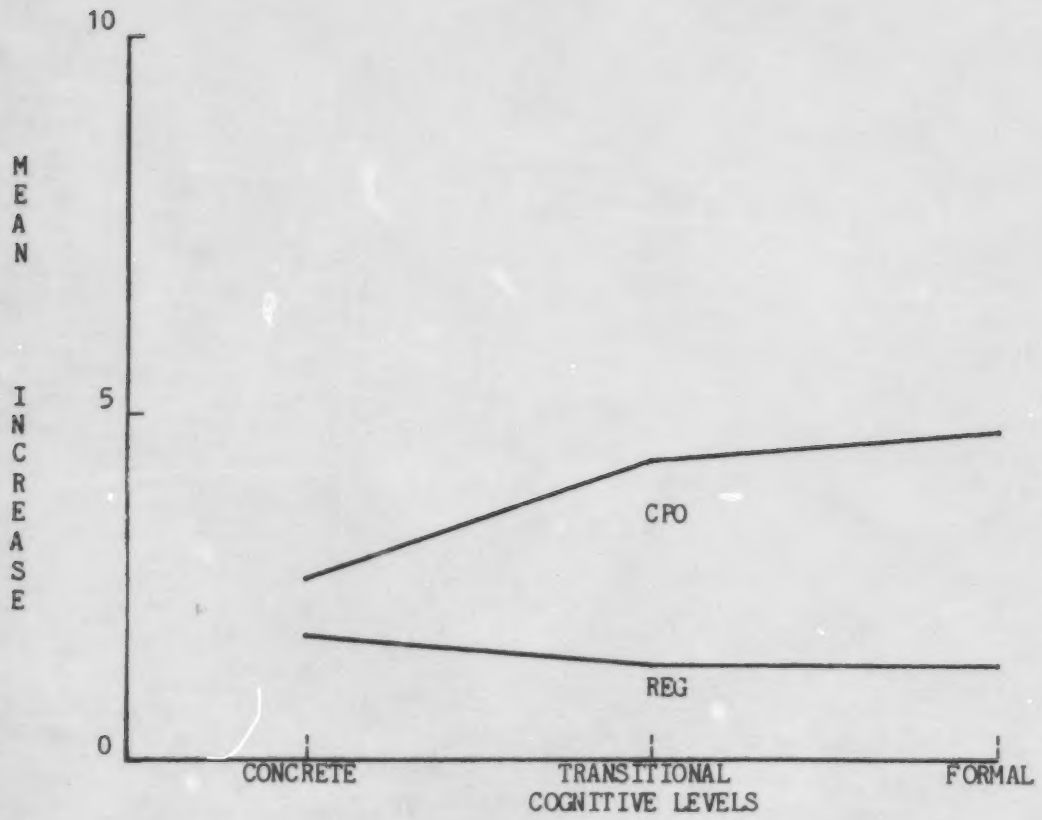


FIGURE 6

MEAN INCREASE IN CSMS RATIO SCORES

BY TREATMENT GROUP AND

COGNITIVE LEVEL

GRADE 7

Grade 8 Achievement: Experimental vs Regular

The Grade 8 achievement test results are presented in Table 4. The pre- and post- test results from the CSMS Fractions test and CSMS Fractions Problem subtest indicate that there were statistically significant differences in favour of the CPO group in the mean measures of improvement (0.05 level). There were also significant differences among the three different cognitive levels with respect to mean improvement scores on the CSMS Fractions test and Fractions Problem subtest. No significant interaction effect between group and level was observed in the CSMS Fractions test improvement scores but significant interaction did occur between group and level in the Fractions Problem improvement scores. A priori t-test comparisons indicated that the CPO formal level mean increases were significantly greater than the REG mean increases at the 0.05 level, but there were no significant differences between the CPO and REG mean increases at the concrete and transitional levels. Figures 7 and 8 represent in graphical form the significant score improvement contrasts found in the Grade 8 achievement data. Accordingly, in the context of fractions, those students at the formal cognitive level improved significantly more than any other level on both the CSMS Fraction test and its Problem subtest.

While there were significant differences among the Fraction Computation improvement scores across the cognitive levels, there was no significant difference between the group mean Fraction Computation improvement scores, nor was there any significant interaction effect between group and level.

The analysis of the Grade 8 CSMS Ratios achievement scores indicated that there were no significant differences between the CPO and REG groups, across the cognitive levels, or attributable to interaction effects.

TABLE 4

CPO VERSUS REG MEAN CONTRASTS WITH PROBABILITY LEVELS FROM REPEATED MEASURES ANALYSES OF VARIANCE ON CSMS FRACTIONS AND SUBSCALES AND CSMS RATIOS SCORES.
GRADE 8

GROUP		CPO			REG			Probability
		Cognitive Levels			Cognitive Levels			
Test or Subtest		Conc.	Tran.	Form.	Conc.	Tran.	Form.	
CSMS Fractions	N	53	37	11	56	82	22	
	Pre-	25.8	41.8	48.2	25.1	38.2	60.1	Group (G) 0.002
CSMS Fractions	Diff.	10.6	12.9	21.7	8.0	11.1	12.5	Level (L) 0.000
(Max = 93)	Post-	36.4	54.7	69.9	33.1	49.3	72.6	G X L Int. 0.153
CSMS Fractions	Pre-	4.7	11.3	14.6	4.3	9.3	21.4	Group (G) 0.003
(Problems)	Diff.	1.1	3.8	9.2	1.8	2.7	3.1	Level (L) 0.000
(Max = 39)	Post-	5.8	15.1	23.8	6.1	12.1	24.5	G X L Int. 0.001
CSMS Fractions	Pre-	2.3	4.4	6.0	2.0	3.3	6.8	Group (G) 0.325
(Computation)		Analysis of Covariance used here						Level (L) 0.004
(Max = 20)	Post-	7.1	9.7	13.2	6.2	9.3	14.9	G X L Int. 0.370
CSMS Ratios	N	45	44	12	39	81	15	
	Pre-	6.6	10.8	18.4	7.2	11.9	20.1	Group (G) 0.563
CSMS Ratio	Diff.	2.4	3.1	3.4	2.1	2.7	1.9	Level (L) 0.446
(Max = 28)	Post-	9.0	3.9	21.8	9.3	14.6	22.9	G X L Int. 0.986

N = Number Tran. = Transitional Int. = Interaction
Conc. = Concrete Form. = Formal

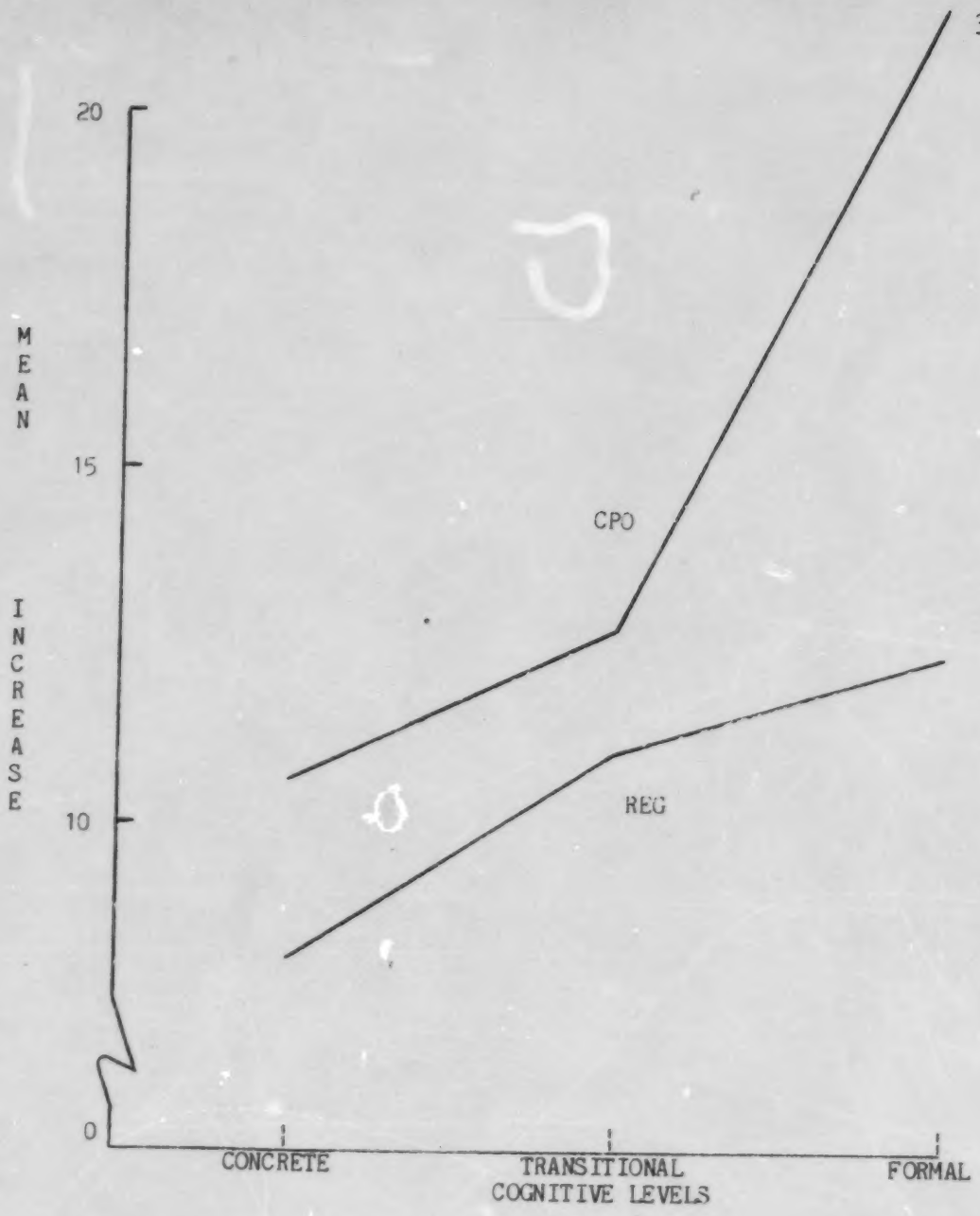


FIGURE 7

MEAN INCREASE IN CSMS FRACTION SCORES

BY TREATMENT GROUP AND

COGNITIVE LEVEL

GRADE 8

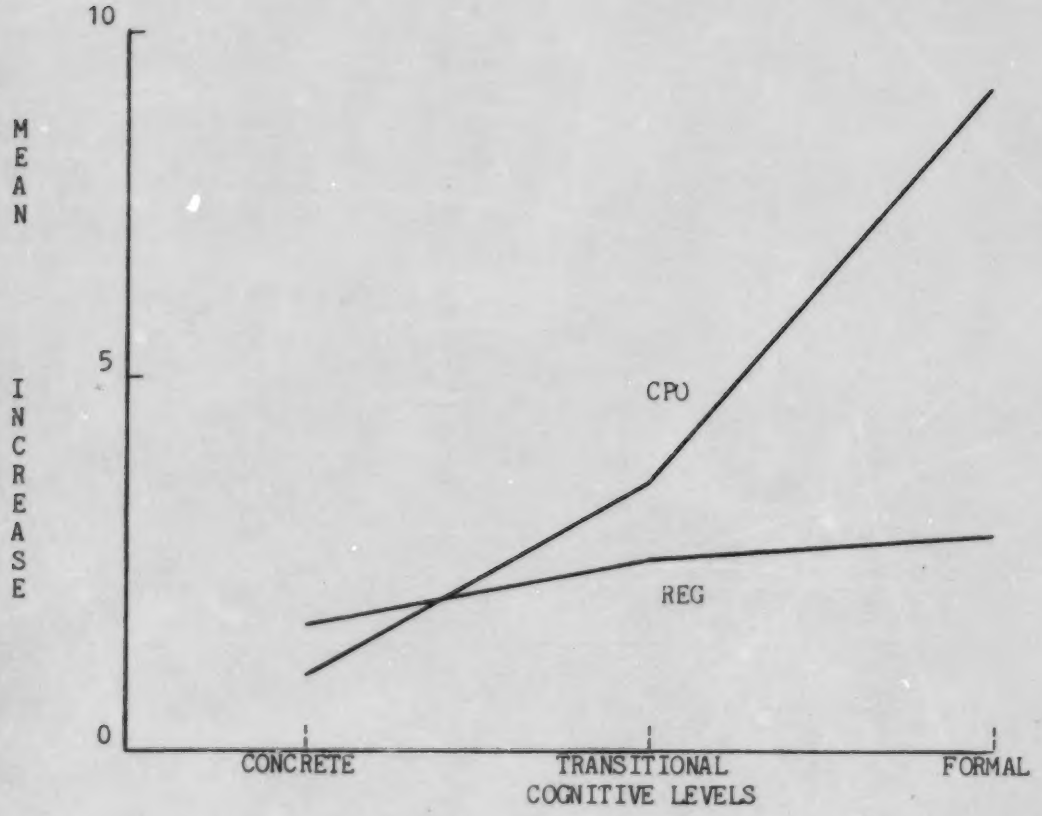


FIGURE 8

MEAN INCREASE IN CSMS PROBLEM SCORES

BY TREATMENT GROUP AND

COGNITIVE LEVEL

GRADE 8

Grade 7 General Mathematical Strategies and Attitude: Experimental vs. Regular

As is shown in Table 5 there were mixed results from Bell's GMS test and its subtests. A significant difference between the mean measures of improvement on Bell's GMS test at the 0.05 level was found favouring the experimental or CPO group. However, for the Generating Examples, Recognizing Relationships and Following Instructions subtests no significant differences between the two groups were found. Nevertheless, the CPO group did show significantly greater improvement than the REG group on the Giving Explanations subtest. The results from the Grade 7 GMS and Attitude analyses are given in Table 5.

The results from the Attitude towards Fractions scale favoured the experimental CPO group, with significant differences being found at the 0.05 level between the group mean improvement measures on the Anxiety and Enjoyment subscales.

Significant differences at the 0.05 level favouring the CPO group on the Anxiety and Enjoyment measures of improvement were found on the Attitude towards Ratios scale, as well.

TABLE 5

CPO VERSUS REG MEAN CONTRASTS WITH PROBABILITY LEVELS FROM REPEATED MEASURES ANALYSES OF VARIANCE ON GENERAL MATHEMATICAL STRATEGIES, FRACTION ATTITUDE, AND RATIO ATTITUDE SCORES - GRADE 7.

Test or Subtest		Max. Score	CPO Mean Score	REG Mean Score	Probability CPO vs REG
General Math Strategies	Pre-	43	11.33 (26%)	12.59 (29%)	0.03
	Post-	43	15.43 (36%)	15.42 (36%)	
Giving Explanations	Pre-	12	2.31 (19%)	2.88 (24%)	0.01
	Post-	12	2.64 (22%)	2.66 (22%)	
Following Instructions	Pre-	6	1.68 (28%)	1.94 (32%)	0.25
	Post-	6	2.44 (41%)	2.48 (41%)	
Recognizing Relationships	Pre-	10	2.59 (26%)	3.56 (36%)	0.08
	Post-	10	4.33 (43%)	4.84 (48%)	
Generating Examples	Pre-	15	4.75 (32%)	4.21 (28%)	0.89
	Post-	15	6.02 (40%)	5.43 (36%)	
Attitude to Fractions Anxiety	Pre-	25	14.13	14.38	0.048
	Post-	25	14.50	13.81	
Attitude to Fractions Enjoyment	Pre-	25	17.60	17.85	0.01
	Post-	25	18.92	17.91	
Attitude to Ratios Anxiety	Pre-	25	14.11	14.25	0.048
	Post-	25	14.25	14.22	
Attitude to Ratios Enjoyment	Pre-	25	17.47	17.66	0.002
	Post-	25	18.74	17.88	

Grade 8 General Mathematical Strategies and Attitude: Experimental vs. Regular

The Grade 8 General Mathematical Strategies (GMS) and Attitude findings are summarized in Table 6. There was no significant difference between the mean measures of improvement in Bell's GMS Total test scores. However, a significant difference in improvement (0.05 level) was found favouring the experimental (CPO) group on the GMS "Giving Explanations" subtest. On the other hand, a 0.05 level significant difference was found favouring the regular (REG) group on the GMS "Following Instructions" subtest. There were no significant differences between the Attitude towards Fractions or Attitude toward Ratios subtest scores (Anxiety and Enjoyment) at the Grade Eight level.

TABLE 6

CPO VERSUS REG MEAN CONTRASTS WITH PROBABILITY LEVELS FROM REPEATED MEASURES ANALYSES OF VARIANCE ON GENERAL MATHEMATICAL STRATEGIES, FRACTION ATTITUDE, AND RATIO ATTITUDE SCORES - GRADE 8.

Test or Subtest		Max. Score	CPO Mean Score	REG Mean Score	Probability CPO vs REG
General Math Strategies	Pre-	43	11.65 (27%)	11.65 (27%)	0.27
	Post-	43	15.22 (35%)	14.47 (34%)	
Giving Explanations	Pre-	12	2.03 (17%)	3.33 (28%)	0.00
	Post-	12	2.66 (22%)	2.49 (21%)	
Following Instructions	Pre-	6	1.69 (28%)	1.34 (22%)	0.035
	Post-	6	2.17 (36%)	2.27 (38%)	
Recognizing Relationships	Pre-	10	3.68 (37%)	3.12 (31%)	0.30
	Post-	10	4.76 (48%)	4.54 (45%)	
Generating Examples	Pre-	15	4.24 (28%)	3.86 (26%)	0.83
	Post-	15	5.63 (38%)	5.17 (34%)	
Attitude to Fractions Anxiety	Pre-	25	13.01	12.73	0.068
	Post-	25	13.08	13.93	
Attitude to Fractions Enjoyment	Pre-	25	17.61	17.27	0.23
	Post-	25	17.68	18.00	
Attitude to Ratios Anxiety	Pre-	25	13.19	13.08	0.89
	Post-	25	13.29	13.25	
Attitude to Ratios Enjoyment	Pre-	25	16.59	17.28	0.24
	Post-	25	17.67	17.72	

DISCUSSION OF FINDINGS

Student Cognitive Levels versus Curricular Demands (Problems 1 to 3, pp. 3,4)

The findings presented in Tables 1 and 2 (pages 14 and 16) demonstrate that, in the contexts of seventh and eighth grade fractions and ratios, significantly greater formal cognitive demands are made on the students by the curriculum guidelines, by textbooks, and by teacher-made tests than is warranted by the fact that only 6 to 12 percent of the students are able to operate formally with fractions and ratios. Similarly, the Teacher Presentations in the regular (REG) classes tended also to be heavily formal (with the exception of one forty minute presentation that was rated entirely transitional). Such premature exposure to formal demands must inevitably lead to rote dependence on algorithmic procedures and/or needless student frustration.

On the other hand, the percentages of class time in which the CPO Teacher Presentations were formal tended to be less than the percentages of formal students (with the exception of grade seven fraction presentations with 18 percent formal demands versus 6 percent formal students - a closer match, however, than was attained by any of the other forms of curricular demand). Similarly, only the CPO Teacher Presentations consistently exhibited significant percentages of the concrete cognitive demands appropriate for the 35 percent to 56 percent of the students unable to operate meaningfully beyond the concrete level. It appears that a significant shift in cognitive demand level away from overly formal approaches has been accomplished with the CPO method. In addition, there is a built-in flexibility in many of the CPO investigations that enables students to find their own level and to work at it, which should have significant practical benefits.

Achievement and Attitude Improvements (Problem 4, p. 4)

The results from the achievement score analyses have shown that, in spite of the many conservative factors involved, the CPO teaching method produced significantly greater achievement gains than the REG method in the teaching of fractions and ratios. Furthermore, at the grade seven level, it was the CPO transitional students, in particular, that showed significantly greater gains than their REG counterparts in fraction test scores. The findings for grade seven ratios were the same, except significant gains favouring the CPO method were also found for the formal students. At the grade eight level, significant differences in fraction test improvement scores were found favouring the CPO group at the formal level, only. For both grades, the significant difference patterns in the fraction problem subtest scores were very similar to those cited for the total fraction test scores. As for fraction computation subtest scores, there were no significant differences between the treatment groups. Accordingly, it appears that the gains made in problem solving facility were not at the expense of computational skill. No significant differences were found between the grade eight CPO and REG ratio achievement scores, which could be attributed to the relative shortness and review character of the grade eight unit.

A significant difference between the two grade seven groups was found on Bell's GMS test favouring the CPO method. The Giving Explanations sub-scale results also favoured the CPO treatment but no significant differences were found on any of the other subscales of Bell's GMS test.

While there was no significant difference between the grade eight groups on the total GMS scores, significant differences were found favouring the CPO group on the "Giving Explanations" subtest and favouring the REG group

on "Following Instructions." One possible explanation for the CPO students not improving significantly on some of the GMS sub tests is that the CPO method was quite different from the teaching method usually employed in local schools. The treatment time may not have been long enough for the sufficient development of the relevant strategies some of which did not even enter significantly into the student investigations that were pursued.

Not only was the CPO method favoured in the context of achievement scores, significant differences were found in grade seven favouring the CPO method in the enjoyment of fractions and ratios. Students in the grade seven experimental classes also demonstrated significantly lower anxiety levels in the context of fractions and ratios. However, at the grade eight level, there were no significant differences between the CPO and REG groups in terms of the Anxiety or Enjoyment scores. Again, one reason that the anxiety level in the grade eight CPO classes was not significantly lowered, and enjoyment not significantly improved, may have been the fact that this was an entirely new way of learning mathematics for the CPO students. As a consequence, they may have been uneasy with the method. The grade eight classes had a relatively short period of time to adapt to "writing-up" about the investigations, a procedure found to be difficult and disliked by many of the students. The ratio unit, which was taught first in the CPO classes, was almost entirely based on investigations. The fraction unit, while striving to be investigational in nature, was not as thoroughly so as the ratio unit was. Such differences in approach combined with the relative shortness and review nature of the grade eight units may have contributed to the different achievement and attitude patterns observed at the two grade levels.

Many reasons could be hypothesized to explain why the CPO group scored higher than the REG group on measures of achievement and attitude (the latter

in grade seven only) but perhaps the main reasons would be the following: in the CPO method attention was paid to the subconstructs of rational numbers (Kieren, 1976, 1977), concrete materials were used, and a time of reflection and discussion was made available to the students. Skemp (1979) would perhaps explain the differences between the two groups' scores in terms of relational learning rather than instrumental learning. In relational learning, second order mental operations are dominant; the goal of this kind of thinking is to relate newly encountered concepts to an appropriate schema. The concrete materials are apparently very helpful to transitional students because using the materials allows the students to begin to bridge the gap between concrete and early formal thinking in rational numbers. The transitional students were able to organize their concepts at a semi-formal level while, at the same time, retaining the support of the concrete material. Judging from the findings of the cognitive levels survey, there are a great many grade seven and eight students who can benefit from the use of such materials.

The fact that there were no significant differences between achievement test scores attained by the concrete level CPO and REG students is not necessarily an indication that concrete level students do not benefit from the CPO method. The achievement tests contained a large proportion of transitional and formal items which, by definition, are beyond concrete operational students, no matter how well they might improve within their level. In any case, it is very important that the concepts, and student confidence in them, be founded on concrete materials for this level of student. Such a foundation should provide future benefits because the student will then have a reliable concrete experience base on which to build. It is interesting to note that the mean increase achieved by the concrete grade

seven CPO and REG students on the CSMS Ratio test shows a trend favouring the CPO group. Since the ratio unit was more investigational in nature than the fraction unit, perhaps these kinds of activities are helpful to the concrete level student. Although the improvement difference between the grade seven CPO and REG formal level students was not statistically significant at the 0.05 level in the context of fractions, there was a definite trend favouring the CPO formal students, perhaps indicating some promotion of relational learning. It could be argued that there are features of the CPO method that promote relational learning, which would be beneficial for students learning to use formal thinking patterns.

CONCLUSIONS

On the basis of the results from the data analyses carried out in the Calgary Junior High School Mathematics Project, answers for the problems posed on pages 3 and 4 can be framed as follows:

1. What levels of cognitive development are demonstrated by students in grades 7 and 8 in the contexts of rational numbers and ratio and proportion?

In a representative sample of over seven hundred grade seven and eight students, the distributions of concrete (C), transitional (T), and formal (F) operational students in the contexts of Fractions (Rational Numbers) and Ratios (Ratio and Proportion) were:

Fractions	Gr. 7 Students	56% C	38% T	6% F
	Gr. 8 Students	45% C	43% T	12% F
Ratios	Gr. 7 Students	50% C	43% T	7% F
	Gr. 8 Students	35% C	54% T	11% F

2. What are the cognitive demands of the present Alberta school mathematics curriculum in the contexts of rational numbers and ratio and proportion as indicated by:

a. the 1978 Alberta Junior High School Mathematics Curriculum Guide

The objectives listed for Fractions (Rational Numbers) and Ratios (Ratio and Proportion) in the 1978 Alberta Junior High School Mathematics Curriculum Guide fell into the following distributions according to cognitive demand:

Fractions	Gr. 7 Objectives	28% C	31% T	41% F
	Gr. 8 Objectives	0% C	27% T	73% F
Ratios	Gr. 7 Objectives	21% C	21% T	57% F
	Gr. 8 Objectives	0% C	20% T	80% F

b. currently authorized textbooks

The explanations and exercises in the introductory sections on Fractions and Ratios in two of the currently authorized grade seven and eight textbooks made cognitive demands that, when averaged for the texts, were distributed in the following way (The individual text cognitive demands distributions are reported in the Appendix):

Fractions	Gr. 7 Text Questions	4% C	24% T	72% F
	Gr. 8 Text Questions	3% C	18% T	79% F
Ratios	Gr. 7 Text Questions	20% C	25% T	55% F
	Gr. 8 Text Questions	1% C	9% T	91% F

c. teacher presentations

Classroom observations of Regular (REG) and Concrete Process-Oriented (CPO) teacher presentations showed that the cognitive demands made by the teachers were distributed as follows:

Fractions	Gr. 7 Teacher Demands	CPO	23% C	59% T	18% F
		REG	0% C	23% T	77% F
	Gr. 8 Teacher Demands	CPO	22% C	75% T	3% F
		REG	0% C	100% T	0% F
Ratios	Gr. 7 Teacher Demands	CPO	90% C	10% T	0% F
	Gr. 8 Teacher Demands	CPO	16% C	77% T	6% F
		REG	0% C	0% T	100% F

d. teacher-made tests

Analyses of the questions posed in Teacher-made Tests produced the following distributions of the cognitive demands made by the test questions:

Fractions	Gr. 7 Test Items	0% C	32% T	68% F
	Gr. 8 Test Items	0% C	2% T	98% F
Ratios	Gr. 7 Test Item	5% C	20% T	74% F
	Gr. 8 Test Items	3% C	5% T	92% F

3. How well do the cognitive demands of selected aspects of the curriculum fit the cognitive abilities of the majority of the students?

In the contexts of grade seven and eight fractions and ratios, it is apparent that the cognitive demands of the Alberta curriculum objectives, the textbooks, teacher presentations, and teacher-made tests do not fit the cognitive abilities of the students very well. However, it is interesting to note that, in general, the "curriculum objectives" cognitive demand distributions appear to reflect the student cognitive level distributions better than the various modes of interpreting the curriculum (but the "objectives" and "student" distributions were, nevertheless, significantly different from one another).

Analyses of the textbooks indicated that 55 percent to 91 percent of the presentations and questions made formal operational demands. However, as reported in answer to problem 1, only 6 percent to 12 percent of the students were able to operate at the formal level with fractions and ratios. The proportion of textbook introductory material on fractions that made only concrete operational demands was 3 percent to 4 percent, but 45 percent to 56 percent of the students were unable to operate beyond that level. In the context of introductory ratios, 20 percent of the material that the grade seven texts provide is at the concrete level (compared with 50 percent concrete operational students), but in the grade eight texts only 1 percent of the material would be appropriate for that 35 percent of the students operating only at a concrete level.

While at least half of the grade seven students were found to be operating at a concrete level, no portions of the teacher presentations

observed in Regular (REG) classrooms were geared to concrete operational thinking. On the other hand, the CPO teacher presentations were concrete 16 percent to 90 percent of the time and formal no more than 18 percent.

None of the teacher-made test items on fractions were posed at a concrete level and no more than 6 percent of the items on ratios were concrete. The proportion of formal fraction and ratio items in the teacher-made tests was of the order of 70 percent in grade seven and 92 percent to 98 percent in grade eight (as compared with approximately 12 percent of the grade eight students who are able to operate formally in the context).

4. Does the use of teaching strategies consistent with the cognitive levels of the students significantly improve their achievement in and attitude towards rational numbers and ratio and proportion topics?

As the cognitive demands analyses have indicated, the Concrete Process-Oriented (CPO) method did provide a range of cognitive demands that, in general, seemed more appropriate for the large proportion of concrete and transitional students than was provided by the Regular (REG) teaching methods.

The statistical analyses of the scores from the pre- and post-test batteries showed that, overall, the CPO students made significantly greater gains in their fraction and ratio achievement test scores than did their REG counterparts. The CPO transitional students appeared to benefit the most from the treatment, followed closely by the formal students. Evidently, for transitional and formal students, the CPO approach consolidates fraction and ratio concepts and skills more effectively than the REG approach. Considering the nature of the CPO

method, it seems reasonable to suggest that not only were there trends (non-significant) favouring the CPO concrete operational students on the achievement measures but the benefits that may have been received could have been masked by the heavy weighting of formal items on the achievement tests (a concrete operational student, by definition, is unable to handle formal tasks). Since there were no significant differences in improvement on computational scores between the CPO and REG groups, the significant achievement and problem-solving gains were apparently not attained at the expense of computational skill.

At the grade seven level there were significant differences favouring the CPO group on Bell's "General Mathematical Strategies" test and on its "Giving Explanations" subscale (on the latter subscale the grade eight results were similar). Presumably, the CPO students' experiences with composing "write-ups" for the investigations and with discussing the investigations with their teachers resulted in improved ability to give explanations. The one apparent anomaly in the findings was the significant difference found in favour of the grade eight REG group on the GMS "Following Instructions" subtest. Significant differences were found in grade seven favouring the CPO group on measures of enjoyment of, and anxiety towards, fractions and ratios. However, at the grade eight level there were no significant differences between the CPO and REG groups on measures of enjoyment and anxiety, perhaps because of the relative shortness and review nature of the grade eight units. It is also possible that student reactions to the write-ups may have had adverse effects.

The CPO method was designed to give students learning experiences

consistent with their level of cognitive development in the contexts of fractions and ratios. Overall, but particularly at the grade seven level, the students taught by the CPO method did show significantly greater improvement in their achievement and attitude towards rational number (fraction) and ratio and proportion topics.

RECOMMENDATIONS

Since it has been demonstrated that a cognitive gap exists between the abilities of students in grades seven and eight and the demands of the Alberta mathematics curriculum, as interpreted by textbooks, teacher presentations, and teacher-made tests, and since it has been demonstrated that a concrete process-oriented teaching approach can effectively improve student achievement, mathematical strategies, and attitudes, it is recommended that some version of the teaching strategies used in this project be made widely available, with suitable inservicing, in the form of alternative approaches that can be inserted in existing programs.

Since the student cognitive development survey coupled with the survey of the cognitive demands of the curriculum and its interpretations have indicated that operations with fractions and development of the concepts and applications of fractions and ratios are not being approached simply enough, even in grade seven and eight, it is recommended that the findings of the study be widely distributed to reinforce present curricular policies indicating that the teaching of operations with fractions be postponed to the seventh and eighth grades (at which time a more concrete approach should be taken than is presently the case).

Since it has been demonstrated that the concrete process-oriented approach can be effectively used by local junior high school teachers within

regular teaching loads and within current topic time restraints, it is recommended that efforts be made to develop and evaluate in classrooms similar approaches to the other topics on the grade seven and eight mathematics curricula (as well as at other grade levels).

LIMITATIONS OF THE STUDY

One limitation of the study described might be the way in which the teachers for the experimental classes were chosen. The teachers were not told what the method or investigation would be but were asked if they were interested in trying out an experimental teaching method. The teachers chosen had expressed an interest in trying a different method of teaching and they were willing to give it a fair trial. Naturally, a related limitation was the inexperience of the teachers with the method. When discussing the teaching of a particular lesson the teachers often felt that they could do a better job the next time. The time that was allowed for using the CPO method was limited and students in the CPO classes had not had any mathematics teaching previously that was similar, judging from comments made by students and teachers and from observations of regular classes. The students did take a few weeks to adjust to the different methods and, concurrently, practical problems that the teachers encounted with the CPO method had to be resolved. An example of this arose with the sloping student desks that the rods and materials slid off of with disrupting regularity.

This study was limited to the teaching of fractions and ratios in seventh and eighth grades, but similar results might well be expected for other topics in the same or other grades.

Although retention testing was planned it could not be completed because of a strike in the Calgary Public School System.

An early version of Bell's GMS test was used for evaluating student ability to give explanations, to follow instructions, to handle relationships, and to generate examples. Bell has indicated that the South Nottinghamshire Project has undertaken revision of this test to refine its characteristics. While the revised test promises to provide improved information, at the time of the investigation the GMS test was the most appropriate instrument available for measuring student ability to use general mathematical strategies.

IMPLICATIONS FOR FURTHER RESEARCH

An implication for further research would be to use the CPO method for a longer time period and for other (if not all) topics in the grade seven and eight mathematics curriculum, perhaps extending it to higher grades as well.

A series of projects could be undertaken in which concrete, process-oriented materials could be developed and evaluated in other topic areas and at various grade levels. It would have been interesting to maintain a longitudinal study of the students in this project to follow their achievement and cognitive development in fractions and ratios.

The results of the NAEP assessment have indicated that adults have difficulty with fraction and ratio problems. An interesting question is whether the difficulty is instructional or cognitive in nature. How could this question best be approached?

Each facet of the secondary mathematics curriculum could be assessed in terms of the relationship between the cognitive demands of the curriculum and the cognitive abilities of the students, with a view to generating an awareness of the need for alternative supplementary teaching approaches.

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APPENDIX

AVAILABILITY OF TESTS USED IN THE CJHMP BUT NOT INCLUDED IN THE APPENDIX TO THE FINAL REPORT

Test	Source
Onslow's "Ratio and Proportion Test"	Onslow, 1976
"CSMS Ratio test" "CSMS Fractions (7th Grade)" "CSMS Fractions (8th Grade)" (Original titles of the tests from which the last two were derived: Fractions, 1st and 2nd year; Fractions, 3rd and 4th year)	Hart, 1981 (sample item analyses and discussion along with large scale test results and patterns). The complete CSMS tests are available from the National Foundation of Educational Research, U.K.
Bell's "General Mathematical Strategies" test (Original title: General Mathematics Test: Finding and Proving Rules)	Bell, 1976 (available from the Shell Research Centre, University of Nottingham, Nottingham, England)

CJHMP COGNITIVE DEMAND LEVELS: FRACTIONS*

CONCRETE (C)

- taking $\frac{1}{1}$, $\frac{1}{2}$ or $\frac{1}{3}$ of a collection of objects, of a drawing or of a number
- doubling, halving or tripling to produce equivalent fractions from a given one
- adding or subtracting using physical models

TRANSITIONAL (T)

- operating with unit fractions with denominator greater than 3 or with fractions for which the numerator and denominator of the basic fraction are no greater than 4
- writing fractions with elements greater than 4 can be considered transitional if pictorial support containing countable parts is provided
- operating with fractions when only doubling or tripling is required for the operation to be carried out (e.g., when generating equivalent fractions)
- dealing with rational number situations beyond the concrete but not formal
- recognizing multiplication and division as inverses

FORMAL (F)

- comparing or operating with fractions in which at least one element is 5 or greater, excluding unit fractions and simple equivalents of $\frac{1}{2}$ or $\frac{1}{3}$
- comparing fractions where relative size has to be judged in terms of units of different size
- requiring second order thinking (e.g., having to simplify before comparing fractions)
- expecting a correct explanation of procedures
- interpreting symbolic representations of fractional relationships

* Derived from the CJHMP Ratio and Proportion Cognitive Demand Levels and from the Fractional Number Thinking Test (Kieren, 1979).

CJHMP COGNITIVE DEMAND LEVELS: RATIO AND PROPORTION

CONCRETE (C) (Lovell, 1971a; Onslow, 1976; Wollman and Karplus, 1974)

- ratios and proportions involving 1 to 2, 2 to 1, 1 to 3, 3 to 1
- solving proportions if only doubling, halving, or tripling are involved or if countable real objects are provided for acting out the proportional relationship.

TRANSITIONAL (T)

- ratios and proportions involving 1 to 4
- solving proportions in which the elements in the basic ratio are no greater than 4
- writing ratios with elements greater than 4 can be considered transitional if pictorial support containing countable parts is provided
- ratio and proportion word problems that do not specify procedures are at least transitional
- ratios and proportions beyond concrete but not formal

FORMAL (F) (Malpas and Brown, 1974; Onslow, 1976)

- solving proportions in which at least one element is greater than 4
- second order proportional thinking required (e. g., having to simplify before comparing ratios)
- interpretation of symbolic representations of proportional relationships
- expectation of a correct explanation using proportional reasoning

TEXTBOOK COGNITIVE DEMANDS ANALYSES

FRACTIONS (RATIONAL NUMBERS)

	Items	Concrete	Transitional	Formal
Holt Mathematics 1 (Gr. 7), pp. 70 to 76	143	3%	7%	90%
Math Is 1 (Gr. 7), pp. 26 to 37	242	4%	34%	62%
Holt Mathematics 2 (Gr. 8), pp. 108 to 114	142	1%	22%	77%
Math Is 2 (Gr. 8), pp. 47 to 55	271	4%	16%	80%

RATIO AND PROPORTION

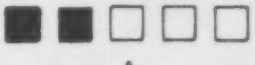
	Items	Concrete	Transitional	Formal
Holt Mathematics 1 (Gr. 7), pp. 90 to 93	72	19%	32%	49%
Math Is 1 (Gr. 7), pp. 170 to 179	177	20%	21%	58%
Holt Mathematics 2 (Gr. 8), pp. 121 to 123	71	0%	3%	97%
Math Is 2 (Gr. 8), pp. 175 to 179	123	1%	12%	87%

FRACTIONAL NUMBER THINKING TEST *

devised by
Thomas E. Kieren
University of Alberta

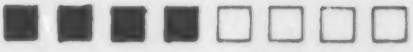
* Item ideas from the work of (with permission) G. Babcock, G. Noelting, A.T. Olson, T. Kieren and B. Southwell. Do not cite without permission from the author.

Items 1 - 6 on this test deal with making an orange drink by mixing orange concentrate and water. In the following items, you are to tell which has the stronger orange flavour mixture: A or B.

SAMPLE:  A B

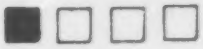
Which has the stronger orange flavour? A Same (B)

Why? *Because B has more orange than water, but A has less.*

1. A B 

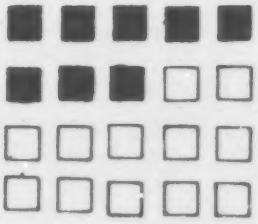
Which has the stronger orange flavour? A Same B

Why?

2. A B 

Which has the stronger orange flavour? A Same B

Why?

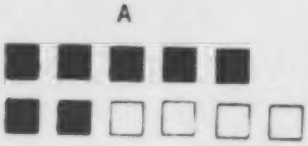
3. A B 

Which has the stronger orange flavour? A Same B

Why?

2.

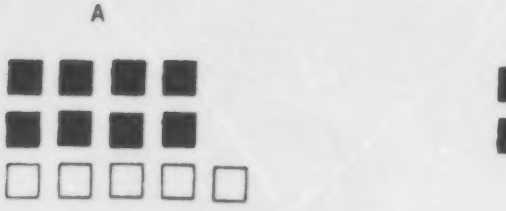
4.



Which has the stronger orange flavour? A Same B

Why?

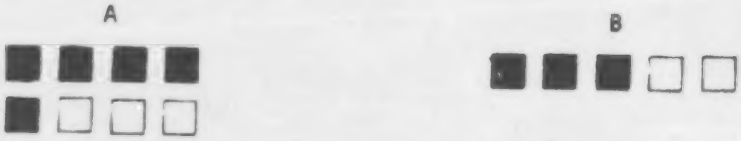
5.



Which has the stronger orange flavour? A Same B

Why?

6.



Which has the stronger orange flavour? A Same B

Why?

Items 7 - 12 are about cookies. Albert (A) is in a group of boys receiving cookies. Betty (B) is in a group of girls receiving cookies. In each item you are to tell who gets more cookies and why.

SAMPLE:

Who gets more? A Same (B)

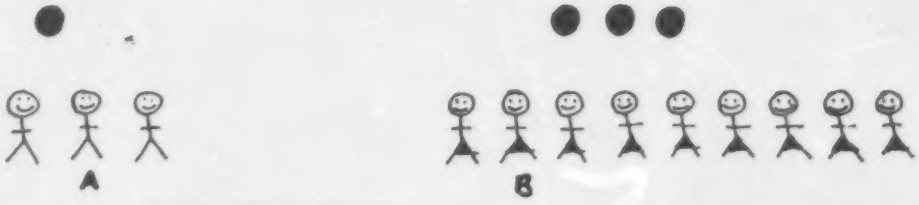
Why? *Well, B gets 2, but A gets less than one.*

7.

Who gets more? A Same B

Why?

8.

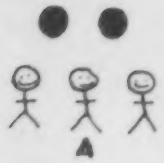


Who gets more? A Same B

Why?

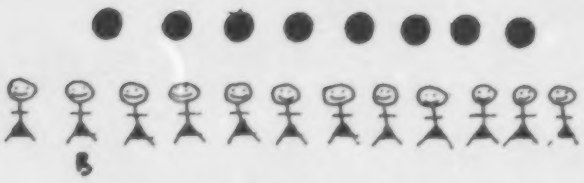
4.

9.



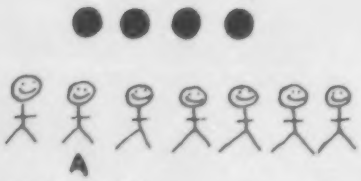
Who gets more?

Why?



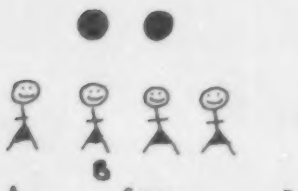
A Same B

10.



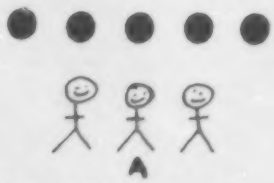
Who gets more?

Why?



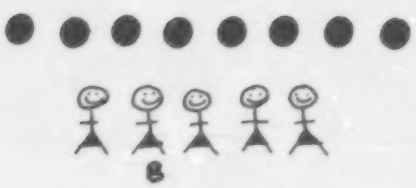
A Same B

11.



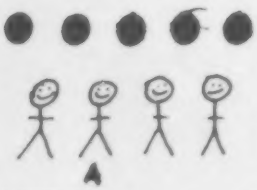
Who gets more?

Why?



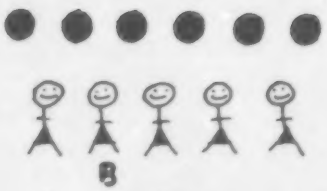
A Same B

12.



Who gets more?

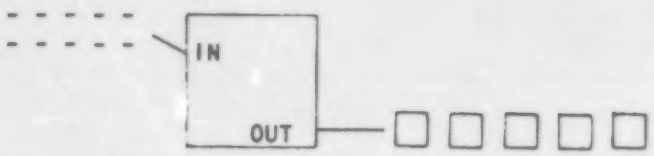
Why?



A Same B

Items 13 - 16 are about machines. Your job will be to figure out how the machines are working and to describe how they work.

SAMPLE: Here is a machine with 10 pieces going in and 5 packs coming out.



Here is some more information.

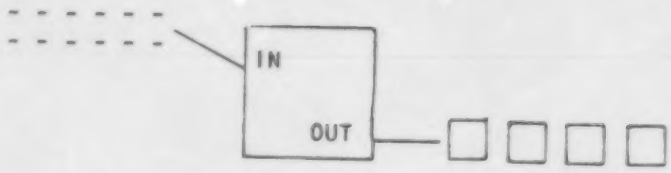
Pieces In		Packs Out
12	→	6
8	→	4
20	→	10

Can you figure these out?

Pieces In		Packs Out
4	→	___
60	→	___
___	→	9

You might have answered 4 → (2), 60 → (30), and (18) → 9

13. Here is another machine. In the picture 12 objects are going in, and 4 packs are coming out.



Here is more information about this machine.

Pieces In		Packs Out
15	→	5
27	→	9
6	→	2
18	→	6

Fill in the following blanks:

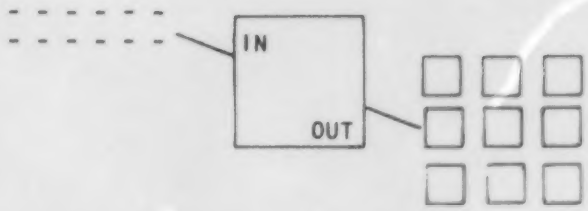
	Pieces In		Packs Out
a)	9	→	____
b)	24	→	____
c)	60	→	____
d)	96	→	____

Describe how you get the number of packs from the number of pieces.

In the items below you are given the number of packs and asked to find the number of pieces.

	Pieces In		Packs Out
e)	____	→	10
f)	____	→	40
g)	____	→	1200

14. Here is another different machine. In the picture when 12 pieces go in, 9 packs come out.



Here is more information about this machine.

Pieces In		Packs Out
20	→	15
8	→	6
28	→	21
44	→	33
60	→	45

Fill in the following blanks:

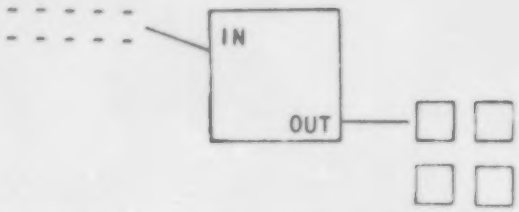
	Pieces In		Packs Out
a)	16	→	____
b)	40	→	____
c)	24	→	____
d)	88	→	____

How do you figure out the number of packs out from a given number of pieces coming in?

Figure out the number of pieces in for a given number of packs.

	Pieces In		Packs Out
e)	____	→	3
f)	____	→	24
g)	____	→	3000

15. Here is a different machine. In this picture, for 10 pieces going in, 4 packs are coming out.



Here is more information about the machine.

Pieces In		Packs Out
30	→	12
25	→	10
100	→	40
45	→	18

Fill in the following blanks.

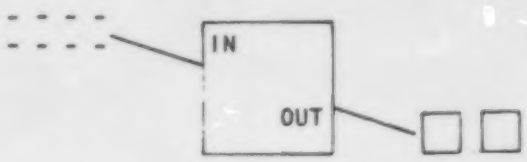
	Pieces In		Packs Out
a)	15	→	____
b)	5	→	____
c)	75	→	____
d)	170	→	____

How do you find the number of packs out from a given number of pieces in?

Find the number of pieces in for the given number of pieces out.

	Pieces In		Packs Out
e)	____	→	14
f)	____	→	8
g)	____	→	2000

16. Here is one more machine. In this picture, 8 pieces are going in with 2 packs coming out.



Here is more information about this machine.

Pieces In		Packs Out
12	→	3
20	→	5
32	→	8
60	→	15

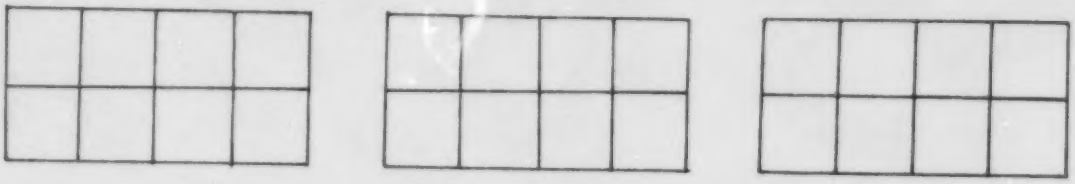
Fill in the following blanks.

Pieces In		Packs Out
16	→	____
4	→	____
100	→	____
15	→	____
M	→	____
____	→	y

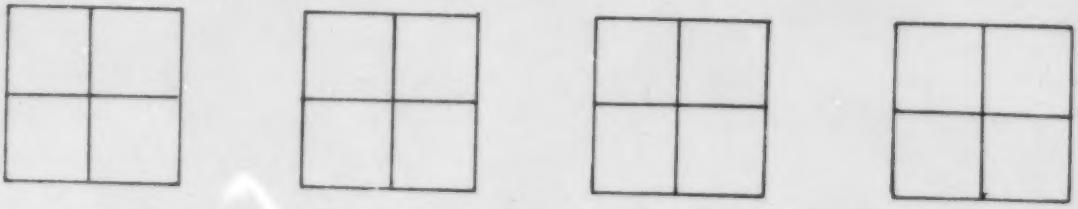
10.

17. Three persons are to share these two medium pepperoni pizzas equally. Shade in one person's share.

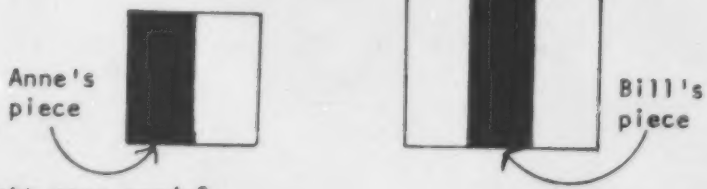
18. Four persons are to share these three chocolate bars equally. Shade in one person's share.



19. Three persons are to share four chocolate bars equally. Shade in one person's share.



20. Anne gets the piece of the bar shown on the left while Bill gets the piece of the bar on the right.



Who gets more candy? Anne Same Bill

Why is this so?

ATTITUDE TOWARDS FRACTIONS (FINAL FORM)

DIRECTIONS: Please fill in your student identification number, school code, name and initials by blackening with an HB pencil the appropriate letter or numbers on the answer sheet provided. Each of the statements on this questionnaire expresses a feeling or attitude towards Fractions. Please indicate, on a five point scale, the extent of the agreement between the attitude expressed in each statement and your own personal feeling. The five points are:
1. Strongly Disagree, 2. Disagree, 3. Undecided, 4. Agree, 5. Strongly Agree.

Mark the answer sheet with an HB pencil giving the best indication of how closely you agree or disagree with the attitude expressed in each statement.

1. Fraction questions are very enjoyable and interesting.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

2. Fraction questions make me feel very uncomfortable, restless and frustrated.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

3. I have a good feeling towards Fraction questions.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

4. When I am faced with Fraction questions I am unable to think clearly.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

5. I have a definite positive feeling towards Fractions and I find them most enjoyable.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

6. Fractions are my most dreaded section in Mathematics.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

7. Fractions are a fascinating section in the mathematics program.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

8. I really get nervous and uptight to even think about doing Fraction questions.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

9. I really like doing Fraction questions.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

10. Having to work out my own solution to Fraction questions makes me feel uncomfortable.
    1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

ATTITUDE TOWARDS RATIOS (FINAL FORM)

DIRECTIONS: Please fill in your student indentification number, school code, name and initials by blackening with an HB pencil the appropriate letter or numbers on the answer sheet provided. Each of the statements on this questionnaire expresses a feeling or attitude towards Ratios. Please indicate, on a five point scale, the extent of the agreement between the attitude expressed in each statement and your own personal feeling. The five points are:
1. Strongly Disagree, 2. Disagree, 3. Undecided, 4. Agree, 5. Strongly Agree.

Mark the answer sheet with an HB pencil giving the best indication of how closely you agree or disagree with the attitude expressed in each statement.

1. Ratio questions are very enjoyable and interesting.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

2. Ratio questions make me feel very uncomfortable, restless and frustrated.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

3. I have a good feeling towards Ratio questions.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

4. When I am faced with Ratio questions I am unable to think clearly.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

5. I have a definite positive feeling towards Ratios and I find them most enjoyable.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

6. Ratios are my most dreaded section in Mathematics.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

7. Ratios are a fascinating section in the mathematics program.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

8. I really get nervous and uptight to even think about doing Ratio questions.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

9. I really like doing Ratio questions.
   1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

10. Having to work out my own solution to Ratio questions makes me feel uncomfortable.
    1. Strongly Disagree 2. Disagree 3. Undecided 4. Agree 5. Strongly Agree

STUDENT QUESTIONNAIRE ON FRACTIONS

DIRECTIONS: Please fill in your name on the answer sheet by blackening the appropriate letters with an HB pencil.

We would like to know how often teachers and students do certain kinds of things in mathematics classes. Please read each question carefully and select the answer that most accurately describes how often each event occurs in your mathematics class.

Record the corresponding numbers 1,2,3,4,5 on the answer sheet using an HB pencil. There are no correct or wrong answers.

1. In our class, students ____ use real objects when they are learning new ideas about Fractions.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

2. In our class, we ____ investigate Fraction problems using real objects.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

3. In answering questions in our class, the teacher ___ uses his/her own examples rather than ours.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

4. We ____ invent our own solutions to Fraction problems and investigations.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

5. Our teacher ____ encourages us to notice number patterns and make guesses at solutions to Fraction problems.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

6. In answering my questions, our teacher ____ refers back to the rules rather than what I did.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

7. In our class, we ____ work together in pairs or groups to solve Fraction problems.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

STUDENT QUESTIONNAIRE ON FRACTIONS

8. We ____ write reports of what we found while working in pairs or groups on Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

9. We are ____ told the rule to use before attempting any problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

10. In our class, the teacher ____ encourages us to use our own methods.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

11. In our class, we ____ discuss the correctness of the rules that we find in group investigations with real objects.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

12. Our teacher ____ gives us rules and several examples of the same type to do when we are starting new work in Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

13. Our teacher ____ encourages us to check the correctness of our method by experimenting with them on other problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

14. If our rule is incorrect our teacher ____ gives us a correct one.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

15. In our class, we ____ meet together to discuss the results we have found in our investigations with Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

16. Our teacher ____ gives us a rule to use for solving new kinds of problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

TABLE 7

T-TEST ANALYSIS OF RESPONSES TO THE STUDENT QUESTIONNAIRE ON FRACTIONS

GRADE 7

QUESTION NUMBER	NUMBER OF RESPONDENTS		MEAN		T-VALUE	P
	CPO	REG	CPO	REG		
1.	207	205	3.11	1.68	15.50	0.00
2.	208	205	3.03	1.58	15.68	0.00
3.	208	207	3.30	3.34	-0.37	0.71
4.	208	206	3.08	2.55	4.29	0.00
5.	208	205	3.14	2.88	2.03	0.04
6.	207	205	3.05	3.01	0.33	0.74
7.	207	207	3.00	1.57	12.78	0.00
8.	206	206	2.44	1.24	12.34	0.00
9.	205	206	3.30	3.88	-4.64	0.00
10.	205	202	2.81	2.22	5.17	0.00
11.	205	203	3.41	2.26	9.28	0.00
12.	205	202	3.72	4.11	-3.67	0.00
13.	205	202	3.45	3.01	3.81	0.00
14.	204	203	3.56	3.77	-1.77	0.08
15.	204	204	3.35	2.44	6.94	0.00
16.	201	203	3.54	3.98	-3.62	0.00

TABLE 8

T-TEST ANALYSIS OF RESPONSES TO THE STUDENT QUESTIONNAIRE ON FRACTIONS

GRADE 8

QUESTION NUMBER	NUMBER OF RESPONDENTS		MEAN		T-VALUE	P
	CPO	REG	CPO	REG		
1.	100	192	2.85	1.77	7.82	0.00
2.	100	192	2.76	1.85	6.55	0.00
3.	100	191	3.27	3.56	-2.17	0.03
4.	100	192	2.83	2.44	2.74	0.01
5.	100	192	3.16	2.68	3.22	0.00
6.	100	191	3.20	3.11	0.68	0.51
7.	100	191	2.21	1.31	7.69	0.00
8.	99	187	2.61	1.27	9.07	0.00
9.	100	189	3.48	3.74	-1.67	0.10
10.	100	188	2.68	2.13	3.96	0.00
11.	100	187	3.32	2.30	7.03	0.00
12.	100	188	3.87	3.68	1.41	0.16
13.	100	188	3.51	2.90	4.30	0.00
14.	100	188	3.57	3.40	1.05	0.30
15.	99	184	3.00	2.21	5.04	0.00
16.	98	182	3.58	3.47	0.72	0.47

STUDENT QUESTIONNAIRE ON RATIOS

DIRECTIONS: Please fill in your name on the answer sheet by blackening the appropriate letters with an HB pencil.

We would like to know how often teachers and students do certain kinds of things in mathematics classes. Please read each question carefully and select the answer that most accurately describes how often each event occurs in your mathematics class.

Record the corresponding numbers 1,2,3,4,5 on the answer sheet using an HB pencil. There are no correct or wrong answers.

1. In our class, students ____ use real objects when they are learning new ideas about Ratios.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

2. In our class, we ____ investigate Ratio problems using real objects.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

3. In answering questions in our class, the teacher ____ uses his/her own examples rather than ours.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

4. We ____ invent our own solutions to Ratio problems and investigations.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

5. Our teacher ____ encourages us to notice number patterns and make guesses at solutions to Ratio problems.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

6. In answering my questions, our teacher ____ refers back to the rules rather than what I did.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

7. In our class, we ____ work together in pairs or groups to solve Ratio problems.
   1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

STUDENT QUESTIONNAIRE ON RATIOS

8. We ____ write reports of what we found while working in pairs or groups on Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

9. We are ____ told the rule to use before attempting any problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

10. In our class, the teacher ____ encourages us to use our own methods.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

11. In our class, we ____ discuss the correctness of the rules that we find in group investigations with real objects.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

12. Our teacher ____ gives us rules and several examples of the same type to do when we are starting new work in Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

13. Our teacher ____ encourages us to check the correctness of our method by experimenting with them on other problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

14. If our rule is incorrect our teacher ____ gives us a correct one.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

15. In our class, we ____ meet together to discuss the results we have found in our investigations with Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

16. Our teacher ____ gives us a rule to use for solving new kinds of problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

TABLE 9

T-TEST ANALYSIS OF RESPONSES TO THE STUDENT QUESTIONNAIRE ON RATIOS

GRADE 7

QUESTION NUMBER	NUMBER OF RESPONDENTS		MEAN		T-VALUE	P
	CPO	REG	CPO	REG		
1.	232	170	3.67	1.79	18.15	0.00
2.	233	169	3.57	1.67	19.17	0.00
3.	232	172	3.25	3.37	-1.09	0.28
4.	231	172	3.19	2.41	5.97	0.00
5.	231	170	3.33	2.82	3.87	0.00
6.	230	171	3.14	2.98	1.37	0.17
7.	231	173	3.68	1.61	17.23	0.00
8.	233	171	3.44	1.31	18.01	0.00
9.	233	171	3.48	3.84	-2.68	0.01
10.	233	168	2.80	2.24	4.79	0.00
11.	232	168	3.66	2.27	10.99	0.00
12.	233	168	3.84	4.05	-1.80	0.07
13.	232	168	3.57	2.76	6.65	0.00
14.	232	169	3.88	3.71	1.31	0.19
15.	231	169	3.36	2.43	6.74	0.00
16.	226	165	3.66	3.93	-2.18	0.03

TABLE 10

T-TEST ANALYSIS OF RESPONSES TO THE STUDENT QUESTIONNAIRE ON RATIOS

GRADE 8

QUESTION NUMBER	NUMBER OF RESPONDENTS		MEAN		T-VALUE	P
	CPO	REG	CPO	REG		
1.	111	117	3.51	1.78	12.04	0.00
2.	110	117	3.44	1.86	10.86	0.00
3.	111	117	3.36	3.56	-1.32	0.19
4.	111	117	3.13	2.15	6.28	0.00
5.	111	117	3.71	2.54	7.40	0.00
6.	109	117	3.32	2.92	2.62	0.01
7.	111	117	3.57	1.47	14.38	0.00
8.	110	114	3.83	1.37	17.54	0.00
9.	109	114	3.61	3.80	-1.12	0.26
10.	110	115	2.73	2.13	4.12	0.00
11.	108	116	3.55	2.28	7.72	0.00
12.	110	115	3.94	3.64	1.80	0.07
13.	110	115	3.53	2.83	4.39	0.00
14.	108	116	3.69	3.45	1.41	0.15
15.	110	114	3.35	2.22	6.49	0.00
16.	108	114	3.79	3.61	1.11	0.27

TEACHING STYLES QUESTIONNAIRE

FRACTIONS GRADE 8

DIRECTIONS: Fill in your name on the answer sheet by blackening the appropriate letters with an HB pencil. There are no correct or wrong answers. Please fill in the school identication code with 998

Record the corresponding numbers 1,2,3,4,5 on the answer sheet using an HB pencil.

Please have one of your Grade 8 mathematics classes complete the accompanying Student Questionnaire. When the student and teacher questionnaires have been completed please seal all questionnaires in the accompanying self-addressed envelope and place it in the School mail.

TEACHING STYLES QUESTIONNAIRE (FRACTIONS GRADE 8)

Thinking of a lesson on Multiplication or Division of Fractions, please indicate on the answer sheet for each item below an estimate of how frequently you would use the teaching style described.

1. I do several examples on the board, carefully and clearly explaining the rules used. Then, I assign similar problems to the students, working with individuals experiencing difficulties.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

2. I explain the rule clearly, do several examples and assign similar problems to the students. Then, I work with individual students having difficulties.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

3. I ask students to read through the appropriate text examples, extracting relevant rules and procedures before they solve the related problems. If the rule discovered is incorrect, I explain the correct rule and assign more problems of a similar kind.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

4. Students work on exercises or problems from the text in small groups while I assist those having difficulties.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

5. I assign students appropriate exercises from the text and help any having difficulty.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

6. Students work in small groups where they investigate mathematical problems with concrete materials. They then discuss with me what they have discovered and write reports on their findings.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

7. I introduce the topic with concrete examples and, where possible, concrete materials. Then I generalize to the mathematical form and follow this with suitable examples and exercises.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

8. Other methods - please describe._____

   _____ _____

   _____

   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

We would like to know how often teachers and students do certain kinds of things in mathematics classes. Please read each question carefully and select the answer that most accurately describes how often each event occurs in your mathematics class.

9. In my class, students ____ use real objects when they are learning new ideas about Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

10. In my class, the students ____ investigate Fraction problems using real objects.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

11. When I answer questions in my class, I ____ use my own examples rather than those of the students.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

12. I ____ encourage my students to invent their own solutions to Fraction problems and investigations.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

13. I ____ encourage my students to notice number patterns and to guess at solutions to Fraction problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

14. When I answer questions, I ____ refer back to the rules I have given rather than what the student did.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

15. In my class, the students ____ work together in pairs or groups to solve Fraction problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

16. I ____ get my students to write reports of what they found while working in pairs or groups on Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

17. I ____ give the rule to use before students attempt problems in Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

18. In my class, I ____ encourage the students to use their own methods, keeping in mind that their methods do not always work.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

19. In my class, the students ____ discuss the correctness of the rules that they discover in group investigations with real objects.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

20. I ____ give the students rules and several examples of the same type to do when they are starting new work in Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

21. I ____ encourage the students to check the correctness of their methods by experimenting with them on other problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

22. If the rule the students use or discover is incorrect I ____ give them the correct one.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

23. In my class, the students ____ meet together to discuss the results they have found in their investigations of Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

24. I ____ give the students a rule for them to use when solving new kinds of problems in Fractions.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

TEACHING STYLES QUESTIONNAIRE

RATIOS GRADE 7,8

DIRECTIONS: Fill in your name on the answer sheet by blackening the appropriate letters with an HB pencil. There are no correct or wrong answers. Please fill in the school identification code with 997 if a 7th grade class is being surveyed or 998 for 8th grade.

Record the corresponding numbers 1,2,3,4,5 on the answer sheet using an HB pencil.

Please have one of your Grade 7 or Grade 8 mathematics classes complete the accompanying Student Questionnaire. When the student and teacher questionnaires have been completed please seal all questionnaires in the accompanying self-addressed envelope and place it in the School mail.

Thinking of a lesson on Equivalent Ratios, please indicate on the answer sheet for each item below an estimate of how frequently you would use the teaching style described.

1. I do several examples on the board, carefully and clearly explaining the rules used. Then, I assign similar problems to the students, working with individuals experiencing difficulties.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

2. I explain the rule clearly, do several examples and assign similar problems to the students. Then, I work with individual students having difficulties.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

3. I ask students to read through the appropriate text examples, extracting relevant rules and procedures before they solve the related problems. If the rule discovered is incorrect, I explain the correct rule and assign more problems of a similar kind.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

4. Students work on exercises or problems from the text in small groups while I assist those having difficulties.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

5. I assign students appropriate exercises from the text and help any having difficulty.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

6. Students work in small groups where they investigate mathematical problems with concrete materials. They then discuss with me what they have discovered and write reports on their findings.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

7. I introduce the topic with concrete examples and, where possible, concrete materials. Then I generalize to the mathematical form and follow this with suitable examples and exercises.
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

8. Other methods - please describe._____

   _____

   _____
   1. Never 2. Infrequently 3. Periodically 4. Frequently 5. Always

TEACHING STYLES QUESTIONNAIRE (RATIOS GRADE 7,8) 3.

We would like to know how often teachers and students do certain kinds of things in mathematics classes. Please read each question carefully and select the answer that most accurately describes how often each event occurs in your mathematics class.

9. In my class, students ____ use real objects when they are learning new ideas about Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

10. In my class, the students ____ investigate Ratio problems using real objects.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

11. When I answer questions in my class, I ____ use my own examples rather than those of the students.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

12. I ____ encourage my students to invent their own solutions to Ratio problems and investigations.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

13. I ____ encourage my students to notice number patterns and to guess at solutions to Ratio problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

14. When I answer questions, I ____ refer back to the rules I have given rather than what the student did.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

15. In my class, the students ____ work together in pairs or groups to solve Ratio problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

16. I ____ get my students to write reports of what they found while working in pairs or groups on Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

17. I ____ give the rule to use before students attempt problems in Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

TEACHING STYLES QUESTIONNAIRE (RATIOS GRADE 7,8) 4.

18. In my class, I ____ encourage the students to use their own methods, keeping in mind that their methods do not always work.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

19. In my class, the students ____ discuss the correctness of the rules that they discover in group investigations with real objects.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

20. I ____ give the students rules and several examples of the same type to do when they are starting new work in Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

21. I ____ encourage the students to check the correctness of their methods by experimenting with them on other problems.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

22. If the rule the students use or discover is incorrect I ____ give them the correct one.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

23. In my class, the students ____ meet together to discuss the results they have found in their investigations of Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

24. I ____ give the students a rule for them to use when solving new kinds of problems in Ratios.
1. Almost Never 2. Seldom 3. Sometimes 4. Often 5. Almost Always

TABLE 11

MEAN RESPONSES OF CPO, REG, AND SURVEY TEACHERS

TO THE FRACTION AND RATIO TEACHING

STYLE QUESTIONNAIRES

GRADES 7 AND 8 COMBINED

TEACHER / QUESTION	FRACTIONS MEAN RESPONSE			RATIOS MEAN RESPONSE		
NUMBER*	CPO 6 TEACHERS	REG 3 TEACHERS	SURVEY 13 TEACHERS	CPO 7 TEACHERS	REG 2 TEACHERS	SURVEY 11 TEACHERS
1.	2.33	4.33	4.23	2.29	4.00	3.64
2.	2.33	3.67	3.85	2.00	4.00	3.45
3.	1.83	2.00	2.15	2.57	1.50	1.91
4.	3.00	1.67	1.92	3.28	1.00	2.55
5.	2.83	3.67	3.61	3.00	3.50	3.82
6.	3.33	1.33	1.69	3.71	1.00	1.45
7.	3.50	2.67	2.92	2.86	2.50	3.36
8.	1.00	2.33	3.46	1.14		2.55
9.	3.67	2.00	1.92	4.29	1.50	1.91
10.	3.17	2.33	1.85	4.00	1.00	1.73
11.	3.17	4.33	3.45	3.00	3.00	3.09
12.	4.00	3.00	3.15	4.00	2.50	3.45
13.	3.50	3.33	3.69	3.29	3.00	3.64
14.	2.67	3.00	3.23	2.86	3.00	3.00
15.	3.50	1.33	1.86	3.43	1.00	2.46
16.	3.33	1.00	1.08	3.17	1.00	1.18
17.	2.16	5.00	3.31	1.57	4.00	3.27
18.	3.00	3.00	2.85	3.57	2.00	3.00
19.	3.33	1.00	1.77	3.71	1.00	1.73
20.	2.17	4.67	4.38	1.57	4.50	3.36
21.	3.17	3.33	3.31	3.86	3.00	3.27
22.	4.00	5.00	4.08	3.34	5.00	3.54
23.	3.00	1.67	2.69	4.00	1.00	1.82
24.	2.20	4.00	3.92	1.86	4.00	3.18

* Teacher Questionnaire question numbers 9 to 24 correspond to Student Questionnaire question numbers 1 to 16.

CLASSROOM OBSERVATION FORM

CALGARY JUNIOR HIGH SCHOOL MATHEMATICS PROJECT

CJHMP CLASSROOM OBSERVATION FORM*

*Constructed around ideas and items drawn from the QUEST Project (Centre for Research in Teaching, U. of A., and Edmonton Public School Board) and the SCAN Project (Shell Research Centre, University of Nottingham, England).

CODES FOR CJHMP CLASSROOM OBSERVATION RECORD

1) GROUPING

I - individuals
P - pairs
SG - small groups
LG - large groups
WC - Whole class

2) METHOD

D - dialogue, teacher to group < 5
E - exposition, teacher to group ≥ 5
I - investigation
IRO - investigation with real objects
RO - real objects being used
DSC - student discussion
SCM - students check and correct methods
SDM - students devise methods
SPH - students find patterns and make hypotheses
SWT - students work on text exercises
REP - students write reports
TCR - teacher corrects student rules
TRE - teacher gives rules and examples
TCW - teacher corrects student work (taking up homework)

3) MODEL OR MATERIALS

CON - concrete
PIC - pictorial
MAN - student manipulatives
DEMO - teacher demonstration
TX - text
WKS - worksheets

4) TQ (teacher questions)

α - recall, single fact, single act, no processing
β - straightforward exercises, putting together several facts or acts
γ - extension of previous work, involving new ideas

5) CD (cognitive demand)

CON - concrete
TRA - transitional
FOR - formal

CJHMP CLASSROOM OBSERVATION RECORD

Teacher _____ Grade, School _____ Year/Month/Day _____

TIME	GROUPING	METHOD	MODEL OR MATERIALS	T Q	COG DEM	COMMENTS
01						
02						
03						
04						
05						
06						
07						
08						
09						
10						
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53						
54						
55						

Observer

CJHMP BEHAVIOUR FREQUENCY RECORD

Teacher | Grade, School | Year/Month/Day

Lesson Type (circle): Introductory, Investigatory, Practice, Problem Solving, Review, _____.

Main Subject Content of Observed Lesson: _____

1. Students used real objects when they were learning new ideas about Fractions or Ratios (RO).

	1	2	3	4	5
N.O.	Low		Med		High

2. Students were enabled to investigate Fraction and Ratio problems using real objects. (IRO)

	1	2	3	4	5
N.O.	Low		Med		High

3. In answering questions, the teacher referred back to rules or his/her own examples. (TRE).

	1	2	3	4	5
N.O.	Low		Med.		High

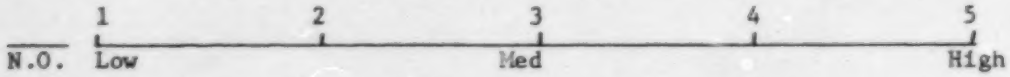
4. Students were encouraged to devise their own methods for attacking Fraction or Ratio problems or investigations (SDM).

	1	2	3	4	5
N.O.	Low		Med		High

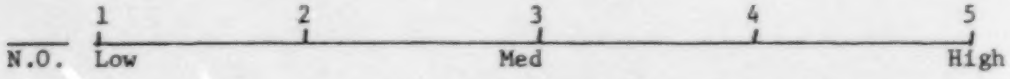
5. Students were encouraged to notice number patterns and/or make guesses at solutions to Fraction or Ratio problems (SPH).

	1	2	3	4	5
N.O.	Low		Med.		High

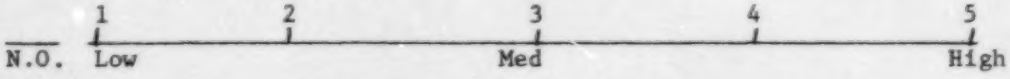
6. Teacher arranged for students to work in pairs or small groups when working on Fraction or Ratio problems or investigations. (P,SG).



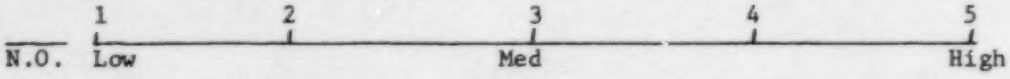
7. Students wrote reports of what they found while working in pairs or groups on Fraction and Ratio investigations (Rep).



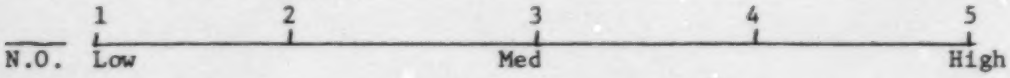
8. Students were given rules and/or examples before attempting any problems or starting new work in Fractions and Ratios (TRE).



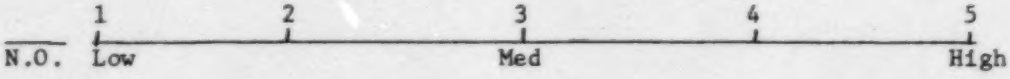
9. The correctness of rules and/or results that students find in group investigations were discussed (Dsc).



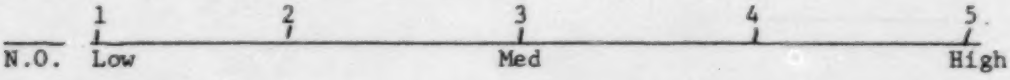
10. Students were encouraged to check the correctness of their methods by experimenting with them on other problems (SCM).



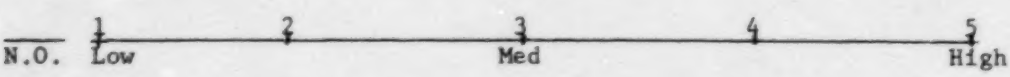
11. Teacher gave students correct rules if theirs were incorrect (TCR).



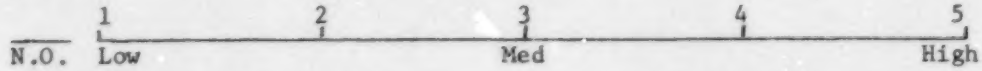
12. Students worked independently or in pairs or small groups on assignments that seemed interesting and worthwhile to them.



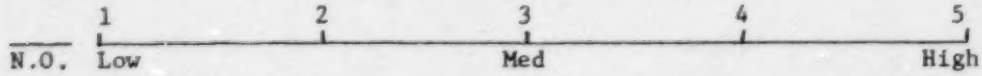
13. Students were enabled to carry out learning tasks with a minimum of direction.



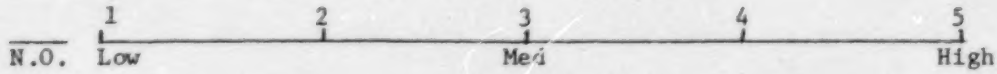
14. Students were actively involved and productively engaged in learning tasks.



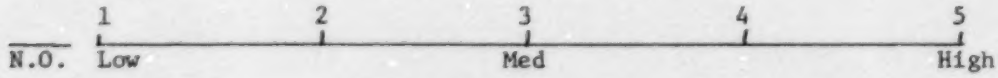
15. The teacher used a variety of instructional techniques, adapting instruction to meet learning needs.



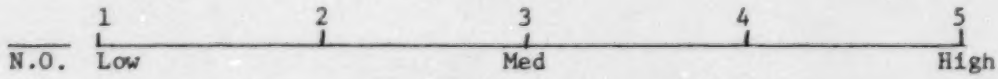
16. Teacher used a system of spot-checking assignments.



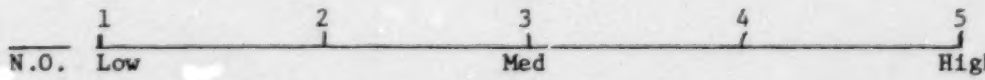
17. Teacher related math games and independent activities and investigations to the concepts to be learned.



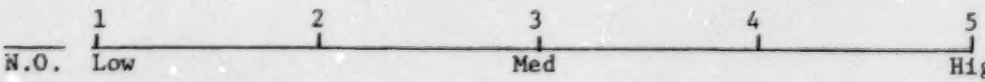
18. Teacher used techniques that provide for the gradual transition from concrete to more abstract activities.



19. Teacher provided evidence of "caring," "accepting," and valuing the student's responses.



20. When students' answers were incorrect or only partially correct, the teacher used techniques such as rephrasing, giving clues, or asking a new question to help the student give an improved response.



21. When students initiated interaction, the teacher accepted and integrated the student question, comment or other contribution.

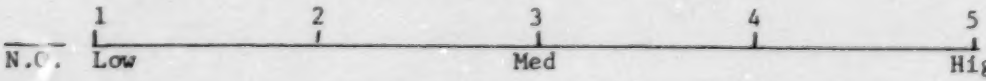


TABLE 12

SUMMARY OF CJHMP CLASSROOM OBSERVATION FINDINGS

AVERAGE PERCENTAGE OF CLASS TIME SPENT IN EACH MODE

GROUPING	Individuals	Pairs	Small Groups	Whole Class
Experimental	35%	18%	8%	39%
Regular	48%	0%	0%	52%

METHOD	Dialog	Expos	Invest	Real Ob	Discus	Text Ex	Cor Work	Other
Experimental	19%	8%	4%	31%	2%	7%	21%	7%
Regular	8%	26%	0%	0%	7%	49%	6%	4%

MATERIALS	Concrete	Pictorial	Demo	Text	Worksheets	Chalkbd	Unclassif
Experimental	31%	7% (man.)	7%	18%	15%	3%	18%
Regular	0%	18% (demo)	13%	48%	21%	0%	0%

TEACHER QUESTIONS	α	β	γ
Experimental	11%	73%	16%
Regular	33%	59%	8%

α : recall; single fact, act; no processing
β : straightforward exercise
γ : extension of previous work

COGNITIVE DEMAND*	CON	TRA	FOR
Experimental	15%	68%	17%
Regular	0%	51%	49%

CON: concrete
TRA: transitional
FOR: formal

* Averaged across Fraction and Ratio lessons; cognitive demand percentage expressed in terms of total time cognitive demands observed (not necessarily whole class periods)

Dialog: Dialogue; Expos: Exposition; Invest: Investigation; Real Ob: Real Objects; Discus: Discussion; Text Ex: Text Exercises; Cor Work: Correcting Work; Chalkbd: Chalkboard; Unclassif: Unclassified.

TABLE 13

EXPERIMENTAL (CPO) AND REGULAR (REG) AVERAGED RATINGS

FROM THE CJHMP BEHAVIOUR FREQUENCY RECORD

Item	CPO	REG	Focus
1	4.4	0.0	real object
2	4.3	0.0	investigation
3	1.5	3.4	teacher rules
4	3.5	1.0	student methods
5	3.75	1.3	number patterns
6	3.4	1.5	small groups
7	3.8	0.0	reports
8	1.5	3.9	prior rules
9	3.0	1.0	discussion
10	3.1	1.3	checking methods
11	1.6	3.5	teacher rules
12	3.6	1.8	small groups
13	3.1	2.3	minimum direction
14	3.5	2.8	involvement
15	3.6	2.7	instructional variety
16	3.7	3.8	spot-checking
17	3.8	1.5	concept activities
18	3.9	1.6	concrete to abstract
19	4.0	3.3	valuing
20	3.6	3.0	response improvement
21	3.8	3.2	contribution acceptance

* Frequency Ratings: 1.0, Low; 3.0, Medium; 5.0, High
(see CJHMP Behaviour Frequency Record

SAMPLE INVESTIGATIONS FROM THE CJHMP

FRACTION AND RATIO UNITS

The original CJHMP teaching unit page numbers have been retained on the following pages. The page numbered 7RP:6 is page 6 from the Grade 7 Ratio and Proportion unit. Similarly, 8RN:31 is page 31 from the Grade 8 Rational Number unit.

PREFACE

The experimental teaching units developed by the Calgary Junior High School Mathematics Project (CJHMP) have been assembled from resource materials that fit an investigative, process-oriented approach to the teaching and learning of mathematics at the Junior High School level. Using these resource materials, the Consulting Pilot Teachers have edited, modified, selected, organized and designed a set of investigations and exercises that cover all of the Ratio and Proportion and Rational Number topics specified for Grades Seven and Eight in the Alberta Junior High School Mathematics Curriculum Guide. They have used the materials with one or more experimental classes as part of their regular teaching load.

By using practical materials as a springboard for initiating mathematical investigations which encourage drawing conclusions and making abstractions and generalizations, the experimental approach provides learning contexts which are appropriate to every level of aptitude in a mixed ability class.

A description of the South Nottinghamshire Project view of learning and teaching mathematics, which has most heavily influenced the design of the CJHMP experimental units, can be found in Journey into Maths*, TG1, pp. iii, iv, 2 to 5, 12 to 17; and TG2, pp. 3 to 5, 7 to 9. Descriptions of how the Number Skills worksheets are intended to be used are given on pages 19 to 21 in TG1 and 11 to 14 in TG2. Copies of these pages as well as the contents pages for TG1 and TG2 are included in the Appendix.

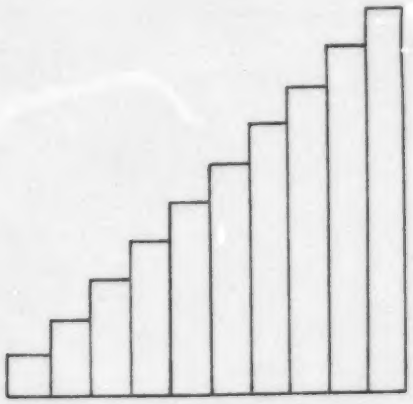
The Grade Seven Experimental Ratio and Proportion Unit has been built almost entirely around South Nottinghamshire Project investigations selected from Journey into Maths, Teacher's Guide 1 and 2 (Bell, Wigley and Rooke, (1978, 1979)).

What follows is a somewhat Canadianized collection of selected student investigation material, activities, and number skill exercises covering the Alberta Grade Seven Ratio and Proportion topics.

*Bell, Alan, Alan wigley and David Rooke, Journey into Maths, Teacher's Guides 1 and 2 (The South Nottinghamshire Project), Bishopbriggs, Glasgow: Blackie and Son, 1978, 1979.

OA 1: RODS (Class Discussion)

You need 1 rod of each colour from the box of Cuisenaire rods. Make a "staircase" with them like this.



How many white rods make a light green?...We say that the value of the light green rod is 3.

What are the values of the other rods?

If we have several black rods and make a "black train" from them, how long could the black train be? (7, 14, 21...)

How long could the yellow train be?

What about other colours?

OA 2: EQUAL TRAINS

Take some light green rods and some yellow rods.

Make a light green train. How long is it?

Can you make a yellow train the same length as your light green train? [Hint: Use Fraction Boards to help keep trains straight.]

Make another light green train. Now can you make a yellow train the same length?

Keep trying until you find a light green train the same length as a yellow train. How long are these equal trains?

Can you find some more equal trains?

How long are the shortest equal trains?

OA 3: MORE EQUAL TRAINS

Take some purple rods and some black rods. Make a purple train. Can you make an equal black train?

Make more trains until you find the shortest equal trains.

Do the same with red rods and light green rods.

Make a table:

Length of 1st colour	Length of 2nd colour	Length of smallest equal trains
3	5	
4	7	
2	3	
5	8	
7	8	
6	9	

Choose more colours.

Try to guess the length of the smallest equal trains.

Can you find a rule?

Write about it.

COMMENTARY FOR OA 1, OA 2, OA 3

Many pupils have not had any previous experience of working with Cuisenaire rods, so that the first activity - a class introduction - is planned as an opportunity for familiarization. Most of them will soon acquaint themselves with the values associated with the different colours, and should then be able to continue with the investigation, starting with OA 2, on their own.

Some pupils may not need the rods, and indeed may prefer to work without them, and there is no reason why they should not be allowed to do so.

All pupils should be encouraged to develop a strategy for finding the shortest equal train, e.g. by starting with one of each colour and always adding to the shortest train until equal trains are found.

Rods having co-prime values are purposely selected initially to assist in spotting a rule. Subsequent selections by the pupils will result in colours with values containing common factors being used, so that the rule has to be modified.

Some may have met the expression 'least common multiple' before, and it can be introduced to the rest at a suitable time.

COMMENTARY FOR OA 4

This is a similar activity with a different slant. The introductory instructions are meant to ensure that, early on, the pupil works with rods having co-prime values, such as 3 and 5 or 4 and 7. Rods with co-prime values a and b allow a train of any length to be made provided that its length is at least $(a-1)(b-1)$, but it is doubtful whether any pupil will spot this. They should, however, discover that it is possible to take trains of any length greater than or equal to a certain value (to be specified in each case). A suitable question to be asked at this point would be how they know that all trains with lengths greater than or equal to this value are possible without actually trying them. Their conclusions are an important part of the work, and the reasoning that they do here is a valuable experience.

Poor colours to choose are ones whose values have a common divisor, and it is worth noting that white is a poor colour because it makes the problem uninteresting!

OA 4 MIXED TRAINS

Take some light green and yellow rods.

Make a "mixed train." You can use either colour on its own, or both colours in the same train.
Record its length.

Make some more.
What lengths can you make?

Make some mixed trains using purple and black.

What lengths can you make now?

Try light green and purple.

Experiment with different colours.
Which colours are good colours to choose?
Can you explain why?

PC 1: Marks

The following marks were obtained by Jane in her end of term tests:

Language Arts	12 out of 20	Science Option	34 out of 55
Science	25 out of 40	Music	28 out of 45
Home Economics	24 out of 50	Math Option	37 out of 55
Social Studies	27 out of 60	French	4 out of 7
Art	24 out of 40	Mathematics	34 out of 50

In which subject did she do best? Which was her worst subject?

Make a new list of subjects and marks to show the order from best to worst.

If you find some subjects difficult to compare, work through PC 2 first.

COMMENTARY FOR PC 1

PC 1 might best be investigated by class discussion.

There may be some initial attempt to compare the subjects using differences, but this is clearly unsatisfactory and an alternative method needs to be found. Much valuable thought can take place in a general discussion concerning performance in given pairs of subjects. Is it better to obtain 25 out of 40 or 25 out of 50? What about 24 out of 50 and 34 out of 50? Or 24 out of 40 and 34 out of 50? During the discussion the idea of comparing ratios needs to be brought out.

Some subjects can be ordered by using approximate numbers; others are too close (Math Option: 37/65 ≈ 40/70, and French: 4/7, are seen to be very close).

When as much ordering as is possible or useful has been done, a discussion on percentages and their use could take place. This might be followed by some practice, using sheets PC 2, PC 5 and PC 6, before returning to this one to complete the ordering, using a calculator where necessary.

Answers

1.	Mathematics	34/50 = 68%
2.	Science	25/40 = 62.5%
3.	Music	28/45 = 62.2%
4.	Science Option	34/55 = 61.81%
5.	Language Arts	24/40 = 60%
6.	Art	12/20 = 60%
7.	French	4/7 ≈ 57.1%
8.	Math Option	37/65 ≈ 56.9%
9.	Home Economics	24/50 = 48%
10.	Social Studies	27/60 = 45%

CJHMP

PREFACE

The experimental teaching units developed by the Calgary Junior High School Mathematics Project (CJHMP) have been assembled from resource materials that fit an investigative, process-oriented approach to the teaching and learning of mathematics at the Junior High School level. Using these resource materials, the Consulting Pilot Teachers have edited, modified, selected, organized and designed a set of investigations and exercises that cover all of the Ratio and Proportion and Rational Number topics specified for Grades Seven and Eight in the Alberta Junior High School Mathematics Curriculum Guide. They have also taught the experimental units to one or more classes as part of their regular teaching load.

By using practical materials as a springboard for initiating mathematical investigations which encourage drawing conclusions and making abstractions and generalizations, the experimental approach provides learning contexts which are appropriate to every level of aptitude in a mixed ability class.

A description of the South Nottinghamshire Project view of learning and teaching mathematics, which has most heavily influenced the design of the CJHMP experimental units, can be found in Journey into Maths*, TG1, pp. iii, iv, 2 to 5, 12 to 17; and TG2, pp. 3 to 5, 7 to 9. Descriptions of how the Number Skills worksheets are intended to be used are given on pages 19 to 21 in TG1 and 11 to 14 in TG2. Copies of these pages as well as the contents pages for TG1 and TG2 are included in Appendices in the CJHMP Grades 7 and 8 Experimental Ratio and Proportion Units.

The material in the Grade Seven and Eight Experimental Rational Number units has been drawn mainly from Journey into Maths, TG2, Dienes' Fractions, an Operational Approach, Kieren's articles on "Constructive Rational Number Tasks," and School Mathematics Project (SMP), Books A & D. Wherever student investigations were not available for a particular topic, those needed were developed using ideas found in the above sources and modelling the investigations after South Nottinghamshire Project procedures.

What follows is a collection of student investigation material, activities, and number skills exercises covering the Alberta Rational Number topics for this grade.

*Bell, Alan, Alan Wigley and David Rooke, Journey into Maths, Teacher's Guides 1 and 2 (The South Nottinghamshire Project), Bishopbriggs, Glasgow: Blackie and Son, 1978, 1979.

FRACTION BOARD*

FB 1: EQUIVALENT FRACTIONS (Class Discussion)

The Fraction Board is designed so that Cuisenaire rods will fit into the six identical slots. Using the fraction board provided, answer the following questions or perform the indicated task.

Fill each of the slots in the board with rods of a single color.

Can you fill all the slots?

The orange rod represents what part or fraction of the whole slot?

The yellow = ?

The purple = ?

The red = ?

The white = ?

How many yellow rods placed together are as long as 1 orange rod?

Therefore $\frac{1}{2}$ = ?

How many red rods placed together are as long as 1 orange rod?

Therefore $\frac{1}{2}$ = ?

Can $\frac{1}{2}$ be written as any other fraction?

Can the purple rod (fifths) be used to find a replacement for $\frac{1}{2}$?

Why or why not?

Find as many fractions as you can that are the same as each of the following (using the fraction board):

$\frac{1}{10}$ = $\frac{3}{4}$ = $\frac{6}{10}$ =

$\frac{1}{5}$ = $\frac{12}{20}$ = $\frac{2}{5}$ =

*Adapted from "The Fraction Board," M. L. Mahaffey, University of Georgia.

FB 2: ADDITION OF FRACTIONS (Class Discussion)

The purple rod is what fraction of the whole slot? $(\frac{1}{5})$

If we place 1 purple rod and 2 more purple rods in the blank slot what fraction of the slot do the three purple rods take up? $(\frac{3}{5})$

How much does $\frac{1}{5}$ and $\frac{2}{5}$ add up to? $(\frac{3}{5})$

How many red rods make up 1 purple rod; 2 purple rods? (2, 4)

So $\frac{1}{5}$ = how many tenths?

$\frac{2}{5}$ = how many tenths?

How would you add up $\frac{1}{5} + \frac{2}{5}$ using tenths (red rods instead of purple)? (Use your fraction board if necessary.)

Write out the sum $(\frac{2}{10} + \frac{4}{10} = \frac{6}{10})$

Is your answer the same as you had before when you used the purple rods?

Now add up $\frac{1}{5} + \frac{2}{5}$ using the white rods. How do you write the sum? $\frac{4}{20} + \frac{8}{20} = ?$

What is the sum using the white rods? $(\frac{12}{20})$

Is the answer the same as before? (yes $\frac{12}{20} = \frac{3}{5}$)

Using your fraction board add up in as many ways as possible:

$$\frac{3}{10} + \frac{4}{10} =$$

$$\frac{1}{4} + \frac{2}{4} =$$

$$\frac{1}{5} + \frac{3}{5} =$$

Can you give a rule for adding fractions? Will your rule always work?

Test your rule on this example:

$$\frac{1}{2} + \frac{1}{4}$$

Check the addition using the fraction board.

FB 2 Continued

What other fractions can $\frac{1}{2}$ be written as? ($\frac{2}{4}$, etc.)

So $\frac{1}{2} + \frac{1}{4}$ can be written as $\frac{2}{4} + \frac{1}{4}$.

Can you add $\frac{1}{5} + \frac{1}{10}$? (use your fraction board)

Can you explain how you got $\frac{3}{10}$?

(i.e. $\frac{1}{5} = \frac{2}{10}$ and $\frac{2}{10} + \frac{1}{10} = \frac{3}{10}$)

Add $\frac{1}{2} + \frac{1}{10}$

$\frac{3}{5} + \frac{3}{10}$

$\frac{1}{20} + \frac{3}{10}$

$\frac{1}{4} + \frac{3}{20}$

Can you give a rule about how to add fractions. Write about it in your binder.

[Further examples may be given but, for now, avoid examples where both fractions must be changed to equivalent fractions. e.g., $\frac{1}{5} + \frac{1}{4}$.]

FB 4: ADDITION AND SUBTRACTION OF FRACTIONS (Class Discussion)

Materials required: 2 sheets of 1 cm dot grid paper per student, 1 overhead 1 cm dot grid transparency for teacher.

McDoodle burgers are put on trays that come in several different sizes. One of the trays holds an arrangement that is 4 burgers wide and 5 burgers long. Now, $\frac{1}{4}$ of the tray are McDoodle's Big M and $\frac{1}{5}$ are McDoodle's Big C burgers. What fraction of the tray would contain Big C or Big M burgers? Let's draw this on the sheets provided. Put a border around the tray. Put an M over the Big M burgers and a C over the Big C burgers. What fraction of the tray has either Big M or Big C burgers? What can you say about the sum of $\frac{1}{4} + \frac{1}{5}$?

When the class goes to McDoodles next time $\frac{1}{4}$ order Big M's and $\frac{1}{3}$ order Big C's. If you wanted to find what fraction of the class are having either a Big M or a Big C, what size tray do you think you would use to solve the problem? Try it on your sheet.

Could you add $\frac{1}{3} + \frac{1}{5}$ this way? Try it out.

Next time the class goes to McDoodles $\frac{3}{5}$ of the class order Big M's with and without cheese. If $\frac{1}{5}$ of the class ordered with cheese, what fraction ordered Big M's without cheese? Could you draw a tray of burgers to show this? What are you actually doing here to solve the problem? $(\frac{3}{5} - \frac{1}{5} = \frac{2}{5})$

If $\frac{1}{3}$ of the class orders Big M's with and without cheese and $\frac{1}{4}$ have ordered with cheese, what fraction ordered without cheese? Draw a "tray" diagram to show this. What subtraction is being done here?
$[\frac{1}{3} - \frac{1}{4} = \frac{1}{12}]$

Could you subtract $\frac{1}{7}$ from $\frac{1}{4}$? Use your "tray" diagram.

[Other examples should follow here and for those students experiencing difficulty a return to FB 2, FB 3 adapted to subtraction of fractions may be helpful.]

Extension: If you are adding $\frac{1}{4} + \frac{1}{6}$ what size "tray" would you need? Can you use a smaller "tray"?

Add $\frac{1}{6} + \frac{1}{15} =$

$\frac{1}{6} + \frac{1}{8}$

Can you find a rule to help find the smallest "tray" diagram you need?

Write about it in your binders.

C.JHMP

7RN: 23a

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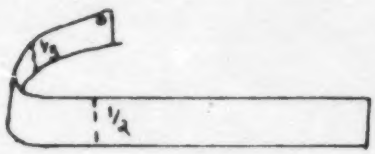
FRACTION TASK 2 :
Meaning of Addition and Measurement

1. Take your tape from task 1. Hold the "1/3" fold directly on the "1/2" fold.

   Where does the "zero" end lie?



   Why?

   A mathematical sentence to describe this is:

   $$1/3 + 1/2 = 5/6$$

2. Repeat 1 above, but fold 1/3 on 7/12. Write the appropriate mathematical sentence.

   $$1/3 + ___ = ___.$$

   Do other "additions" using your tape.

   Can you "add" fractions without like denominators?

3. What happens if you lay the "2/3" fold on the "5/6" fold?

   Can you figure out how much beyond 1 the tape extends?

   Complete this mathematical sentence

   $$2/3 + 5/6 = 1 ____$$

4. Using the tape, do other additions whose sum is greater than 1. Write the related mathematical sentences.

5. Using your tape (and imagination) to solve the following:

   $1/3 + ____ = 5/6$ $\quad\quad$ $____ + 1/12 = 11/12$ $\quad\quad$ $5/8 + 1/2 = ____$

6. Think up a way to show subtraction using your tape.

T. E. Kieren, "Constructive Rational Number Tasks", Delta-K, Edmonton, Mathematics Council of the A.T.A., March 1979, pp . 31, 32.

MULTIPLICATION OF FRACTIONS

Objectives

While working through these activities, pupils should make progress in developing the following concepts and skills (content objectives):

(a) know how to act out a multiplication of fractions calculation using pictorial or concrete aids;

(b) establish the notion that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Pupils should also make progress in developing the following general strategies (process objectives):

(a) use pictorial or concrete embodiments to aid learning

(b) apply mathematics to life-like situations

Introduction

Materials required: 2 student worksheets, several sheets of 1 cm dot grid, O/H projectual of 1 cm dot grid, Cuisenaire rods.

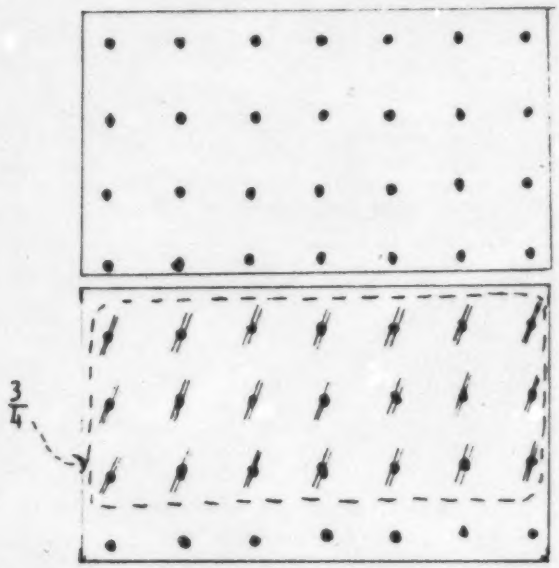
MF 1, 2 consis[illegible] a class introduction to multiplication of fractions using arrays followed by individual or pair activities which are somewhat exploratory and intended to give some insight into and to lead to generalizations about the process of multiplying fractions. Some pupils may only need to do a few examples, but care should be taken to ensure that hasty conclusions are not drawn from limited experience. Supplementary questions like "what would $\frac{a}{b} \times \frac{c}{d}$ be for any numbers a, b, c, and d?" could give an indicator of how well the idea has been generalized. If a greater number of examples is required, these could be provided by the teacher or, preferably, made up by the pupils themselves. Those students whom the teacher is satisfied have a sound understanding of the ideas could proceed to work in Number Skills 2 (perhaps an investigation or problem set) until the whole class is ready to move on to the next topic.

MF 3, is designed for similar purposes to those of MF 1, 2 but is built around Cuisenaire rods rather than arrays. They could be used to supplement or to replace MF 1, 2 at the teacher's discretion.

The ideas for the MF activities have been strongly influenced by Dienes' Fractions, an Operational Approach (arrays) and Trivett's And So On (rods).

MF 1: MULTIPLICATION OF FRACTIONS (Class Discussion)

[Each student will need two sheets of 1 cm dot paper. Draw a line around a 4 x 7 array of dots on the O/H.]



Draw a line around an array of dots just like this on your dot paper to represent a grade eight class of 28 students.

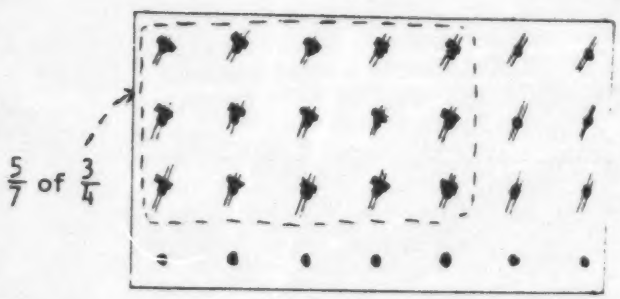
Three-quarters of the students in this class know how to ski. Show each skier by drawing short parallel lines on either side of a student dot. [. . . a bird's eye view of a skier!] [Some suggested leading questions: What does it mean to take $\frac{3}{4}$ of something? How can four equal parts of the class be easily described? How many of the four equal parts are skiers? How many skiers are there?]

We could record the process of finding $\frac{3}{4}$ of 28 as:

$\frac{3}{4}$ of 28 OR $\frac{3}{4} \times 28 = (28 \div 4) \times 3$

$= .7 \times 3$

$\frac{3}{4} \times 28 = 21$



Five-sevenths of the skiers are planning to go on the school ski trip. Draw a little narrow rectangular pack sack on the backs of enough skier dots to show how many skiers are planning to go on the trip. How many students is that? [15] [The group of skiers has to be divided into how many equal parts? How many of those parts are planning to go on the ski trip?]

MF 1 (Continued)

How would that be recorded? . . . This time we have taken $\frac{5}{7}$ of 21.

$$\frac{5}{7} \text{ of } 21 \text{ OR } \frac{5}{7} \times 21 = (21 \div 7) \times 5$$
$$= 3 \times 5$$
$$\frac{5}{7} \times 21 = 15$$

The skiers going on the trip make up what fraction of the whole class? Don't simplify, just for now. $[\frac{15}{28}]$

In other words:

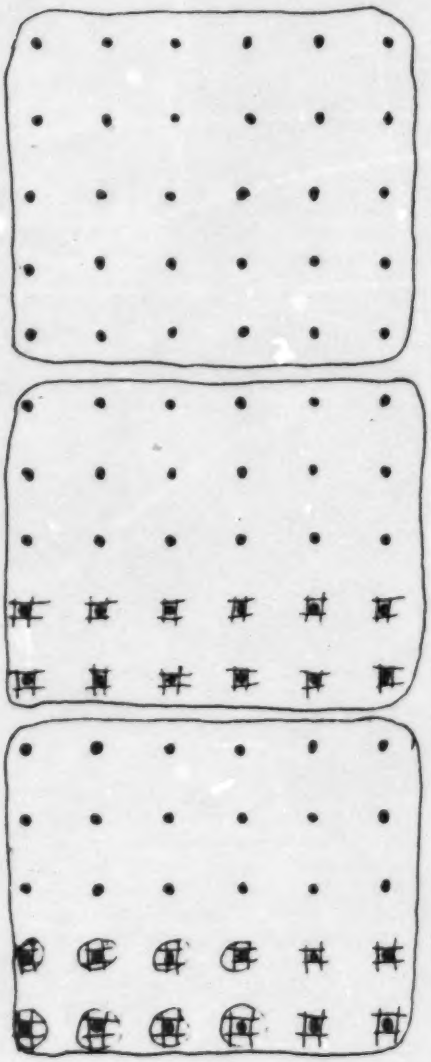
$$\frac{5}{7} \text{ of } \frac{3}{4} \text{ OR } \frac{5}{7} \times \frac{3}{4} = \frac{15}{28}$$

CJHMP 8RN: 34

MF 2: MULTIPLICATION OF FRACTIONS (Examples)

You will need a sheet of 1 cm dot paper.

Here is an example of what we have been doing.



In a class of 30 students a poll showed that $\frac{2}{5}$ of the students like to play tic-tac-toe in free time. Of those students, $\frac{4}{6}$ also like to play checkers. What fraction of the class likes to play tic-tac-toe and checkers?

To represent the class, we can draw a line around a 5 x 6 array of dots (5 x 6 because that gives the number of students in the class and can be divided by both 5 and 6).

The $\frac{2}{5}$ of the class that play tic-tac-toe can be shown by marking 2 of the 5 equal rows of dots with #'s--12 students.

The $\frac{4}{6}$ of the tic-tac-toe players who also play checkers can be shown by marking 4 of the 6 equal columns of tic-tac-toe players--8 students.

$\frac{8}{30}$ of the class like to play tic-tac-toe and checkers.

We have found that $\frac{4}{6}$ of $\frac{2}{5}$ of the class is $\frac{8}{30}$ of the class.

$\frac{4}{6}$ of $\frac{2}{5}$ OR $\frac{4}{6} \times \frac{2}{5} = \frac{8}{30}$

Exercises

Try these examples. You might like to make up a story for some of the examples and invent your own dot symbols for coding the different rows or columns of dots to fit the exercise and story. In each case, choose an array that can be divided evenly by both denominators. Do not simplify your fraction answers until you are sure of the pattern for multiplying two fractions.

1. $\frac{3}{5} \times \frac{2}{7}$ 2. $\frac{2}{3} \times \frac{2}{5}$ 3. $\frac{3}{7} \times \frac{2}{5}$ 4. $\frac{7}{9} \times \frac{3}{10}$ 5. $\frac{4}{3} \times \frac{2}{5}$

6. $\frac{1}{3} \times \frac{2}{3}$ 7. $\frac{5}{4} \times \frac{3}{7}$ 8. $4 \times \frac{1}{2}$ 9. $\frac{3}{4} \times \frac{5}{9}$ 10. $4 \times \frac{2}{5}$

Make a record of your findings:

	Fractions Multiplied	Product
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		

Write a rule in your binder that tells how to multiply two fractions.